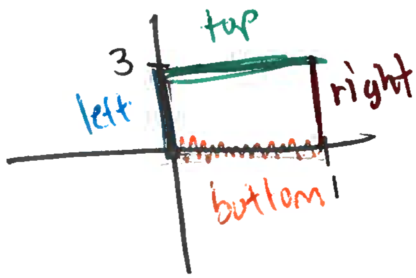
Continued from 21 Sept

$\hookrightarrow f(x,y) = y^2 + xy - 4x^2$



last time: c.p. at $(0,0)$ check

Top : max at $x = 3/8 \Rightarrow (3/8, 3)$

left : $x=0,\ 0 \le y \le 3$

$\Rightarrow f(0,y) = y^2$

$\frac{df}{dy} = 2y \overset{\text{set}}{=} 0 \rightarrow \boxed{\text{min at } y=0} \Rightarrow (0,0)$

$\frac{d^2f}{dy^2} = 2 > 0$

bottom : $y=0,\ 0 \le x \le 1$

$f(x,0) = -4x^2$

$\frac{df}{dx} = -8x \overset{\text{set}}{=} 0 \rightarrow \boxed{\text{max at } x=0} \rightarrow (0,0)$

$\frac{d^2f}{dx^2} = -8$

no work!

$(1, -\frac{1}{2})$

right : $x=1,\ 0 \le y \le 3$

$f(1,y) = y^2 + y - 4 \quad \frac{df}{dy} = 2y + 1 \overset{\text{set}}{=} 0 \rightarrow \boxed{\text{max at } y = -\frac{1}{2}}$

$\frac{d^2f}{dy^2} = 2$

OK now check results

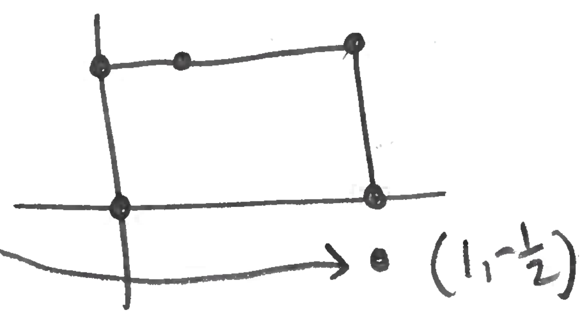
Points (x,y)	$f(x,y)$
$(0,0)$	0
$(\frac{3}{8},3)$	$\frac{153}{16} \approx 9.56$
$\cancel{(1,-\frac{1}{2})}$	$\cancel{-\frac{17}{4}}$
$(0,3)$	9
$(1,0)$	-4
$(1,3)$	8

← abmax at $(\frac{3}{8},3)$ w/ value 9.56

← ~~abmin at $(1,-\frac{1}{2})$ w/ value $-\frac{17}{4}$~~

NOT IN Rectangle

← abmin

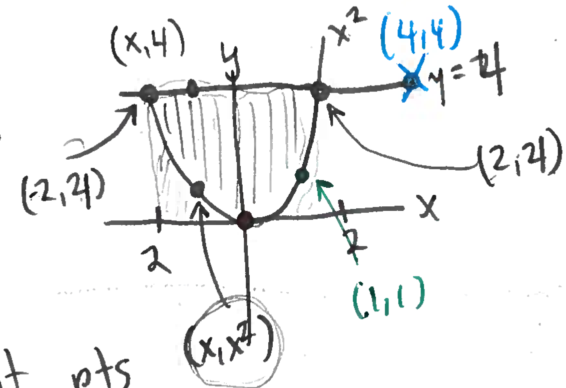


Ex: Find abs extrema of

$$f(x,y) = x^2 - 2xy + 2y$$

on domain bounded between $y = x^2$ and $y = 4$.

Soln:



Crit pts

$$\begin{cases} f_x = 2x - 2y \overset{\text{set}}{=} 0 & (i) \\ f_y = -2x + 2 \overset{\text{set}}{=} 0 & (ii) \end{cases}$$

$(ii) \rightarrow x = 1$

$\downarrow (i)$

$2 - 2y = 0 \rightarrow y = 1$

$\Rightarrow \boxed{(1,1) \text{ is a c.p.}}$

Boundary

line $y=4$: $y = 4$, $-2 \le x \le 2$

$$g(x) = f(x,4) = x^2 - 8x + 8$$

$$\frac{dg}{dx} = 2x - 8 \overset{\text{set}}{=} 0 \rightarrow x = 4 \Rightarrow \boxed{(4,4)}$$

NOT considered

on $y = x^2$: $h(x) = f(x,x^2) = x^2 - 2x^3 + 2x^2 = 3x^2 - 2x^3$

$\cancel{x^2(3-2x) = 0}$

$\cancel{\boxed{(0,0)} \leftarrow x = 0}$ $\quad \cancel{x = 3/2}$

$$h'(x) = 6x - 6x^2 \overset{\text{set}}{=} 0$$

$$x(6-6x) = 0$$

$x = 0 \rightarrow \boxed{(0,0)}$

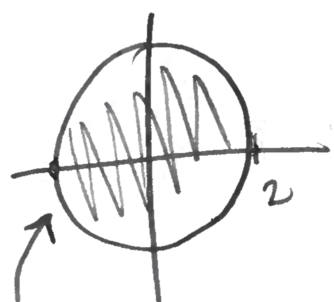
$x = 1 \rightarrow \boxed{(1,1)}$

Points (x,y)	$f(x,y)$
$(1,1)$	1
$(0,0)$	0
$(-2,4)$	28
$(2,4)$	-4

$28 \leftarrow$ abmax occurs at $(-2,4)$

$-4 \leftarrow$ abmin occurs at $(2,4)$

$x^2+y^2 \leq 4$



bounding circle

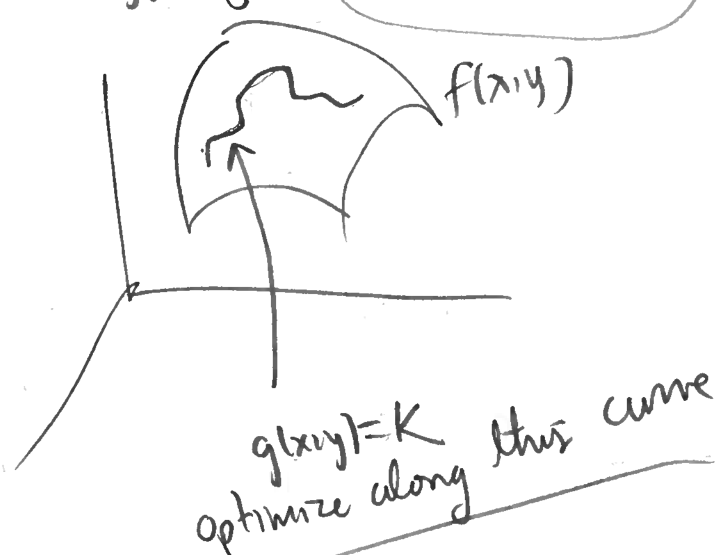
$x^2+y^2=4$

$y = +\sqrt{4-x^2}$ $\quad$ $y = -\sqrt{4-x^2}$

Lagrange Multipliers

earlier: [we maximized $V = xyz$
subject to constraint
$2xz + 2yz + xy = 12$]

Lagrange multipliers help us find extreme values of $f(x,y)$ given $g(x,y) = K$



$2x + y = 1$

Method – to find min & max values of $f(x,y,z)$ subject to $g(x,y,z) = K$

(1) Find all x, y, z and λ such that
a number

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = K \end{cases}$$

(2) evaluate f at all pts found
largest → max
smallest → min