Matrix determinants

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad \frac{d}{dx}(f) = \frac{df}{dx}$$

2×2 3×3

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

Ex: $\det\left(\begin{bmatrix} 3 & 1 \\ -2 & 5 \end{bmatrix}\right) = 15 - (-2) = 17$

3x3 determinants

$$\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = a\det\left(\begin{bmatrix} e & f \\ h & i \end{bmatrix}\right) - b\det\left(\begin{bmatrix} d & f \\ g & i \end{bmatrix}\right) + c\det\left(\begin{bmatrix} d & e \\ g & h \end{bmatrix}\right)$$

$$= a(ei - fh) - b(di - gf) + c(dh - eg)$$

<u>Ex</u>: $\det\left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\right)$

$$= 1\det\left(\begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix}\right) - 2\det\left(\begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix}\right) + 3\det\left(\begin{bmatrix} 4 & 5 \\ 7 & 8 \end{bmatrix}\right)$$

$$= 1(45-48) - 2(36-42) + 3(32-35)$$

$$= -3 + 12 - 9$$

$$= 0$$

<u>Ex</u>: $\det\left(\begin{bmatrix} 1 & 3 & 0 \\ 5 & 1 & 6 \\ 2 & 1 & 8 \end{bmatrix}\right)$

$$= 1(2) - 3(28) + 0$$

$$\begin{array}{r} {}^{2}28 \\ 3 \\ \hline 84 \end{array}$$

$$= 2 - 84$$

$$= -82$$

Cross Product

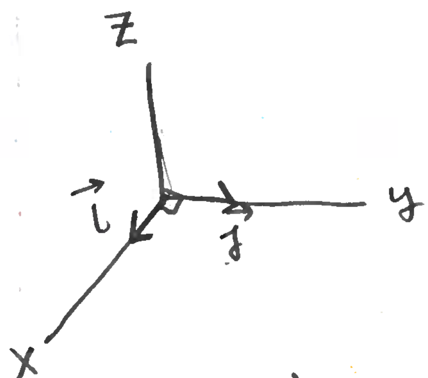
$\vec{x} \perp \vec{y}$ iff $\vec{x} \cdot \vec{y} = 0$

1. must have vectors in $\mathbb{R}^3$
2. the cross product is perpendicular to both vectors being multiplied

Let $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$

$$\vec{a} \times \vec{b} := \det\left(\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}\right)$$

Ex:



$\vec{i} = \langle 1, 0, 0 \rangle$

$\vec{j} = \langle 0, 1, 0 \rangle$

Compute $\vec{i} \times \vec{j} = \det\left(\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\right)$

$$= \vec{i} \det\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) - \vec{j} \det\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) + \vec{k} \det\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$= \langle 1, 0, 0 \rangle (0) - \langle 0, 1, 0 \rangle (0) + \langle 0, 0, 1 \rangle (1)$$

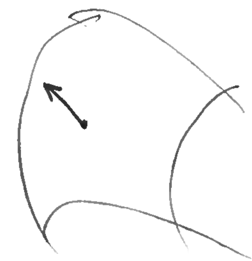
$$= \underline{\langle 0, 0, 1 \rangle} = \vec{k}$$

Ex: $\vec{j} \times \vec{i} = \det\left(\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\right)$

$= \vec{i}(0) - \vec{j}(0) + \vec{k}(-1)$

$= -\vec{k}$

$\Rightarrow$ conclude cross product is not commutative!!



Ex: $\langle 1,2,3 \rangle \times \langle -1,2,8 \rangle$

$= \det\left(\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ -1 & 2 & 8 \end{bmatrix}\right)$

$= \vec{i}(10) - \vec{j}(11) + \vec{k}(4)$

$= \langle 10, -11, 4 \rangle$

Show $\langle 10,-11,4 \rangle \perp \langle 1,2,3 \rangle$:

$\langle 10,-11,4 \rangle \cdot \langle 1,2,3 \rangle = 10 - 22 + 12 = 0$

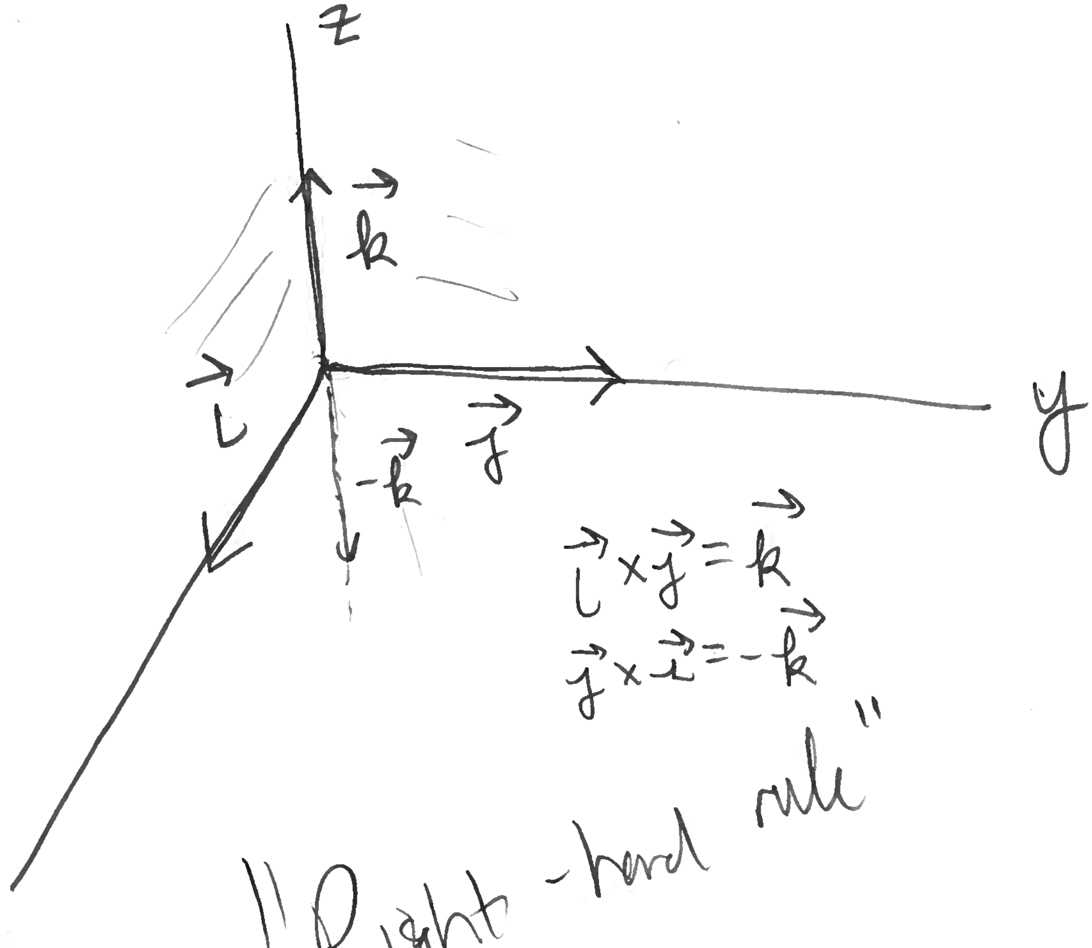
Show $\langle 10,-11,4 \rangle \perp \langle -1,2,8 \rangle$:

$\langle 10,-11,4 \rangle \cdot \langle -1,2,8 \rangle = -10 - 22 + 32 = 0$

Cross products are anti-commutative

$$\vec{x} \times \vec{y} = -(\vec{y} \times \vec{x})$$

How do we know what direction the cross product points?



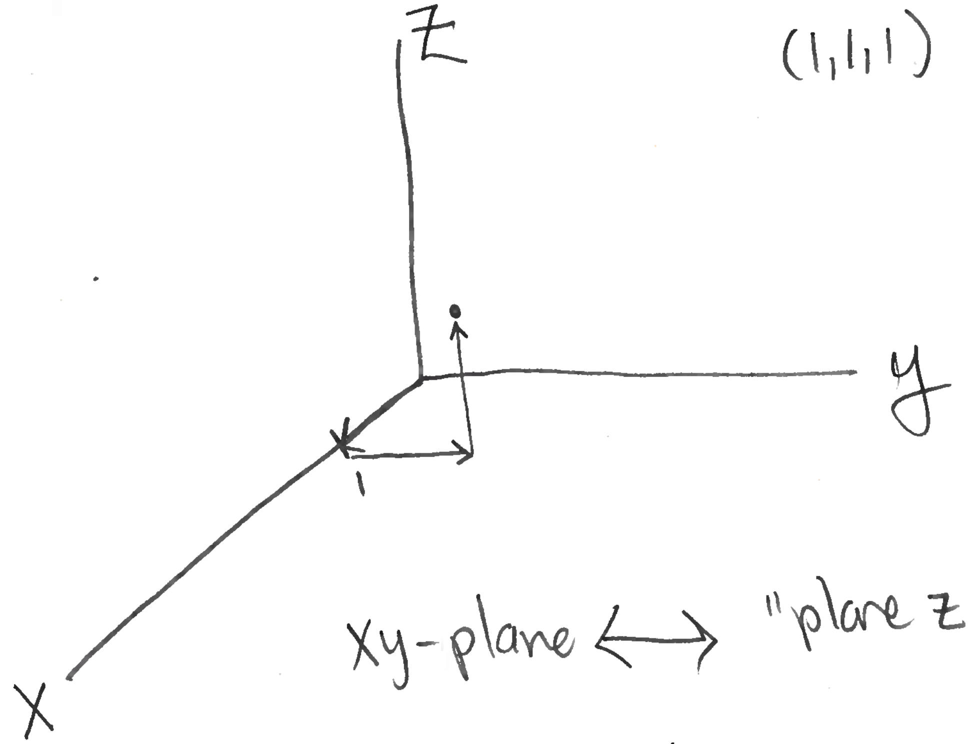
$\vec{i} \times \vec{j} = \vec{k}$

$\vec{j} \times \vec{i} = -\vec{k}$

"Right-hand rule"

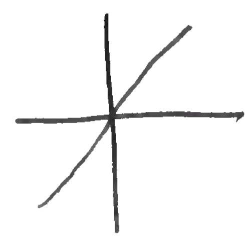
Eqts of lines + planes in space

3D coordinate system

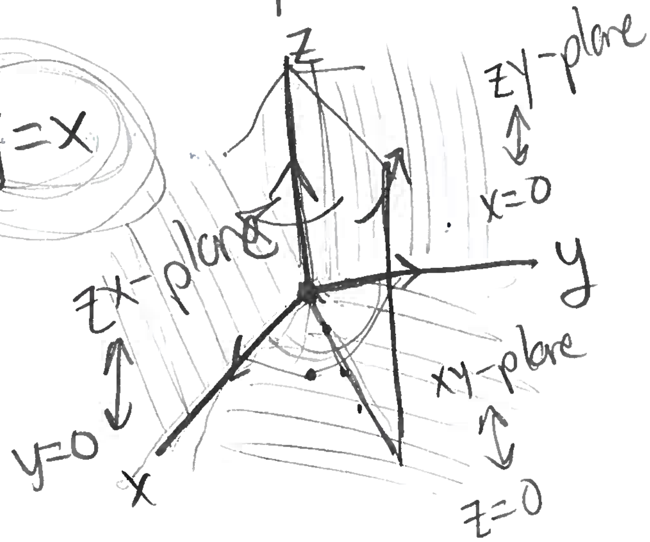


xy-plane $\longleftrightarrow$ "plane $z=0$"

$y=x$



$y=x$



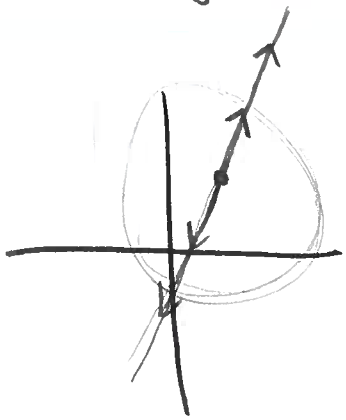
Lines

earlier

point + slope

$\downarrow$

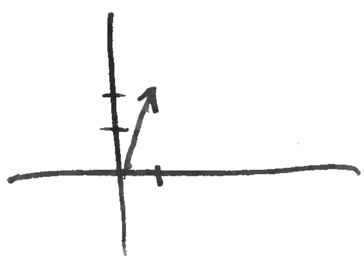
eqt of line



line of slope 2

$\updownarrow$

$\langle 1, 2 \rangle$



now

point, vector

$\downarrow$

eqt of line