Written HW14 – MATH 2501 Fall 2020

Due by 11 November for timely completion credit

In this problem, you will solve the basic differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(t). \tag{1}$$

If we (technically, illegally) separate the numerator and denominator of the derivative, we can write

$$\mathrm{d}y = y(t)\mathrm{d}t$$

and dividing by $y(t)$ would yield

$$\frac{\mathrm{d}y}{y(t)} = \mathrm{d}t$$

If we squint our eyes (trust me here), we can think of this equation as

$$\frac{y'(t)}{y(t)} = 1 \tag{2}$$

1. Let $f(x)$ be an unknown function and calculate $\dfrac{\mathrm{d}}{\mathrm{d}t}\ln(f(t))$. Your answer should include both $f(t)$ and $f'(t)$.

2. Use your result above to determine the antiderivative(s) of $D^{-1}\left(\dfrac{f'(t)}{f(t)}\right)$.

3. Use your result above to find the antiderivative(s) (with respect to the variable t) of both sides of equation (2) (*note: take $f(t) = y(t)$ here*).

4. Solve the equation resulting from the previous problem for the function $y(t)$. What you have just found is called "the general solution of the differential equation (1)".

5. Consider the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(t), \quad y(1) = 15.$$

   Use your answer to the previous question to solve this initial value problem by finding the unknown constant.