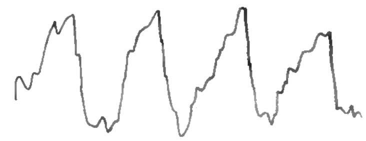
Summation notation

Ex: Compute $\displaystyle\sum_{l=2}^{5} (l+1)$

end value

start value

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$



Soln:

$$\sum_{l=2}^{5} (l+1) = \underset{(l=2)}{(2+1)} + \underset{(l=3)}{(3+1)} + \underset{(l=4)}{(4+1)} + \underset{(l=5)}{(5+1)}$$

$$= 3+4+5+6$$

$$= 18$$

Ex: Compute

$$\prod$$

$$\sum_{m=5}^{8} e^{m^2+1} = e^{5^2+1} + e^{6^2+1} + e^{7^2+1} + e^{8^2+1}$$

$$= e^{26} + e^{37} + e^{50} + e^{65}$$

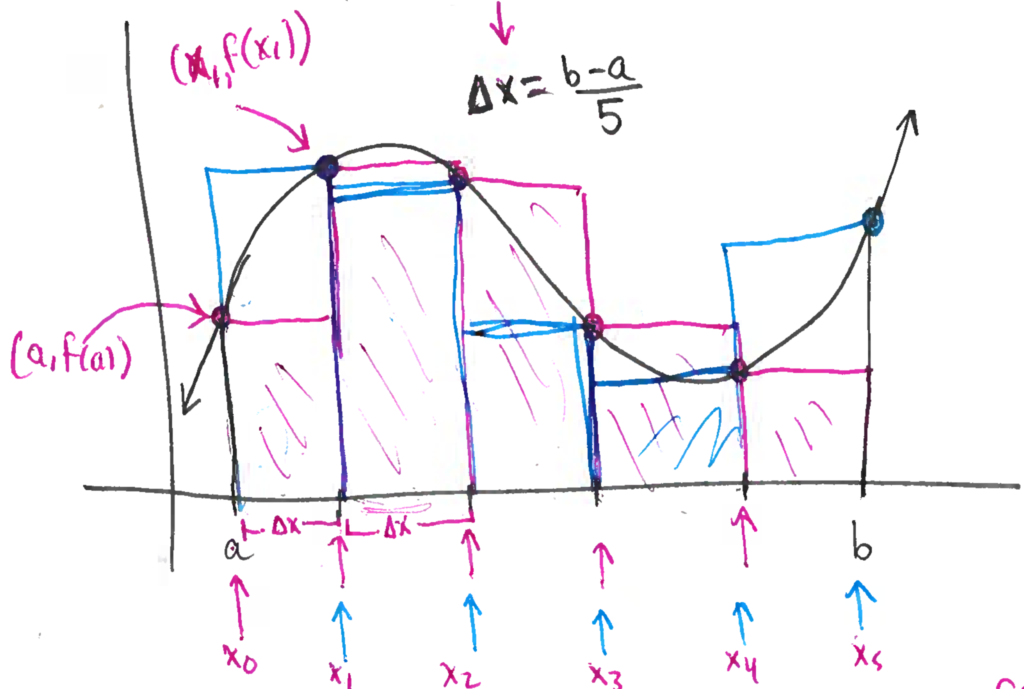
Right vs Left Riemann sums

Left Riemann sum in pink

Right Riemann sum in blue

Use $n=5$ rectangles.

$$\Delta x = \frac{b-a}{5}$$



Left RS $= f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$

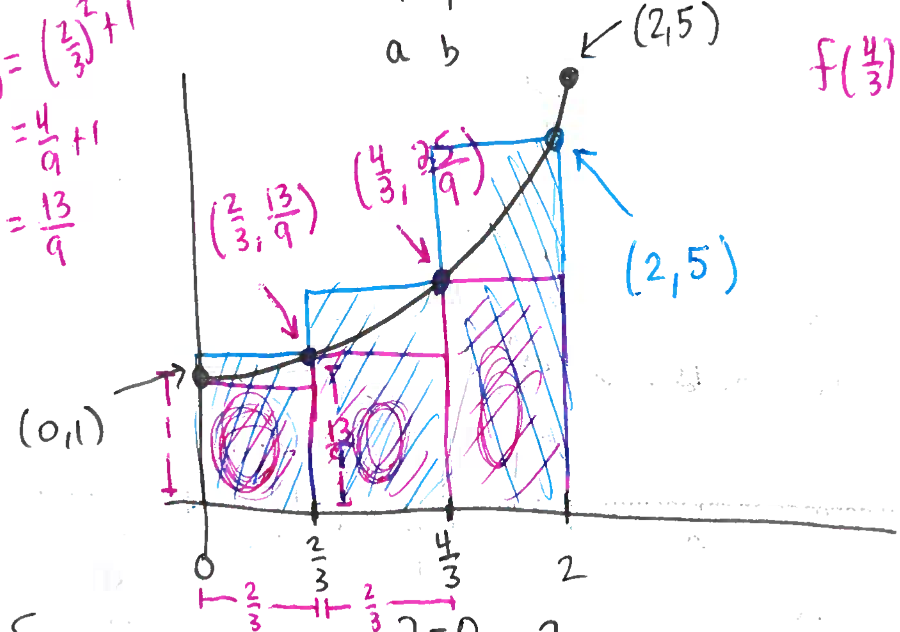
right RS $= f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x + f(x_5)\Delta x$

Ex: Left + right Riemann sums for $f(x) = x^2 + 1$ on $[a,b] = [0,2]$ with $n=3$ rectangles.

$$[a,b] = [\underset{\substack{\uparrow \\ a}}{0}, \underset{\substack{\uparrow \\ b}}{2}]$$

$$f\left(\tfrac{2}{3}\right) = \left(\tfrac{2}{3}\right)^2 + 1 = \tfrac{4}{9} + 1 = \tfrac{13}{9}$$

$$f\left(\tfrac{4}{3}\right) = \left(\tfrac{4}{3}\right)^2 + 1 = \tfrac{16}{9} + 1 = \tfrac{25}{9}$$

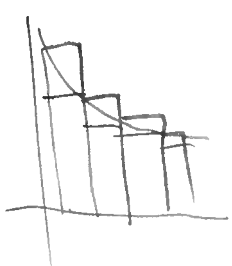


Since $n=3$, $\Delta x = \dfrac{2-0}{3} = \dfrac{2}{3}$

$$\text{left RS} = (1)\left(\tfrac{2}{3}\right) + \left(\tfrac{13}{9}\right)\left(\tfrac{2}{3}\right) + \left(\tfrac{25}{9}\right)\left(\tfrac{2}{3}\right) = \tfrac{94}{27} \approx 3.48$$

$$\text{right RS} = \left(\tfrac{13}{9}\right)\left(\tfrac{2}{3}\right) + \left(\tfrac{25}{9}\right)\left(\tfrac{2}{3}\right) + (5)\left(\tfrac{2}{3}\right) = \tfrac{166}{27} \approx 6.14$$

$$3.48 < \underbrace{\int_0^2 x^2 + 1 \, dx}_{\frac{8}{3} + 2 = \frac{14}{3} \approx 4.67} < 6.14$$



$f(x) = x^2+1$

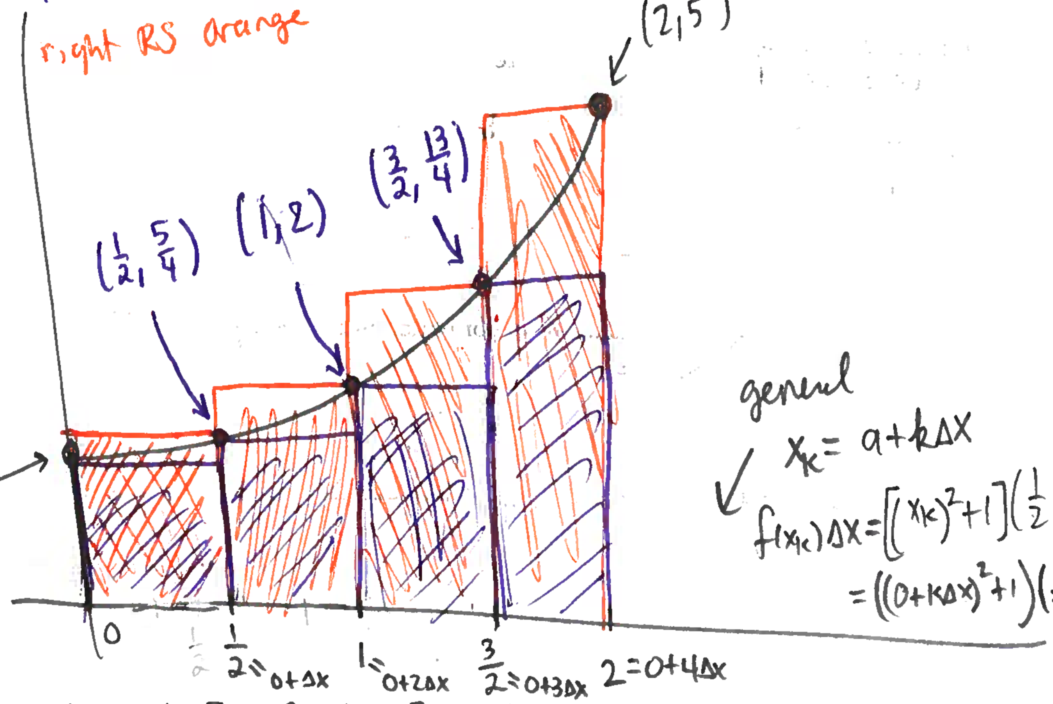
Ex: Same as last time but with $n=4$

left RS purple

right RS orange

$f(\frac{1}{2}) = (\frac{1}{2})^2+1$
$= \frac{1}{4}+1$

$f(\frac{3}{2}) = (\frac{3}{2})^2+1$
$= \frac{9}{4}+1$



general
$x_k = a+k\Delta x$
$f(x_k)\Delta x = [(x_k)^2+1](\frac{1}{2})$
$= ((0+k\Delta x)^2+1)(\frac{1}{2})$

Since $n=4$ and $[a,b]=[0,2]$: $\Delta x = \frac{2-0}{4} = \frac{2}{4} = \frac{1}{2}$

left RS $= (1)(\frac{1}{2}) + (\frac{5}{4})(\frac{1}{2}) + 2(\frac{1}{2}) + (\frac{13}{4})(\frac{1}{2}) = \frac{15}{4} \approx 3.75$

right RS $= (\frac{5}{4})(\frac{1}{2}) + (2)(\frac{1}{2}) + (\frac{13}{4})(\frac{1}{2}) + (5)(\frac{1}{2}) = \frac{23}{4} = 5.75$

$3.48 < 3.75 < \underbrace{\int_0^2 x^2+1\,dx}_{\text{area under curve} = 4.67} < 5.75 < 6.14$