$$D^{-1}x^n = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1), \quad D^{-1}\left(\frac{1}{x}\right) = \ln(x) + C, \quad D^{-1}\left(e^{ax}\right) = \frac{1}{a}e^{ax} + C$$

Ex: $$D^{-1}\left(\frac{3}{\sqrt{x}} + \sqrt[3]{x}\right) = D^{-1}\left(3x^{-1/2} + x^{1/3}\right)$$

$$= 3D^{-1}\left(x^{-1/2}\right) + D^{-1}\left(x^{1/3}\right)$$

$$= 3\left(\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right) + \frac{x^{\frac{1}{3}+1}}{\left(\frac{1}{3}+1\right)} + C$$

$$= \left(\frac{3}{\left(1/2\right)}\right)x^{1/2} + \frac{x^{4/3}}{4/3} + C$$

$$= 6\sqrt{x} + \frac{3}{4}\underbrace{\sqrt[3]{x^4}}_{x\sqrt[3]{x}} + C$$

$\sqrt{x} = x^{1/2}$

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

$\sqrt[3]{x} = x^{1/3}$

$\frac{1}{\sqrt{x}} = x^{-1/2}$

$\frac{1}{3} + 1 = \frac{1}{3} + \frac{3}{3}$

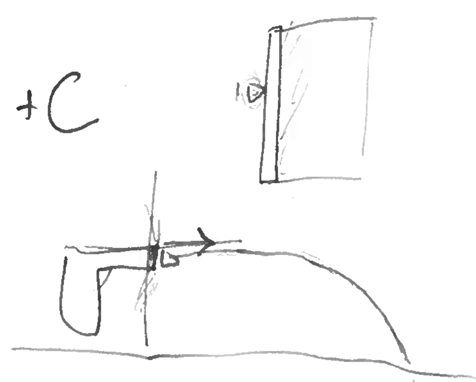
Ex: Calculate $F = D^{-1}f$ where $f(x) = x^2 + 3x$ equipped with initial condition $F(0) = 5$.

Soln: $$F = D^{-1}f = D^{-1}\left(x^2 + 3x\right) = \frac{x^3}{3} + \frac{3x^2}{2} + C$$

$$5 \underset{\substack{\uparrow \\ \text{given}}}{=} F(0) \underset{\substack{\uparrow \\ \text{computed from my formula}}}{=} \cancelto{0}{\frac{0^3}{3}} + \cancelto{0}{\frac{3(0)^2}{2}} + C$$

$$\boxed{5 = C}$$

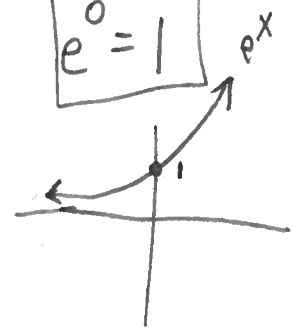
Therefore, $$\boxed{F(x) = \frac{x^3}{3} + \frac{3x^2}{2} + 5}$$



Ex: Calculate $F = D^{-1}f$ where $f = e^{2x} + e^{-3x}$ equipped with initial condition $F(0) = 15$.

Soln: Calculate

$\boxed{e^0 = 1}$



$$F = D^{-1}\left(e^{2x} + e^{-3x}\right)$$

$$= D^{-1}\left(e^{2x}\right) + D^{-1}\left(e^{-3x}\right)$$

$$= \frac{1}{2}e^{2x} - \frac{1}{3}e^{-3x} + C$$

3
15
6
90

$$15 \underset{\substack{\uparrow \\ \text{given}}}{=} F(0) \underset{\substack{\uparrow \\ \text{calculated}}}{=} \frac{1}{2}e^{2(0)} - \frac{1}{3}e^{-3(0)} + C$$

$$15 = \frac{1}{2}(1) - \frac{1}{3}(1) + C$$

$$15 = \frac{1}{6} + C$$

$$C = 15 - \frac{1}{6} = \frac{90}{6} - \frac{1}{6} = \frac{89}{60}$$

Therefore,

$$\boxed{F(x) = \frac{1}{2}e^{2x} - \frac{1}{3}e^{-3x} + \frac{89}{60}}$$

Ex: Calculate $F = D^{-1}f$ where $f = \frac{1}{2x} + e^{3x} + x^5$ equipped (3)
with initial condition $F(3) = 12$.

Soln: Calculate

$$F = D^{-1}f = D^{-1}\left(\frac{1}{2x} + e^{3x} + x^5\right)$$

$$= \frac{1}{2}\ln(|x|) + \frac{1}{3}e^{3x} + \frac{x^6}{6} + C$$

$$12 \underset{\text{given}}{=} F(3) \underset{\text{calculate}}{=} \frac{1}{2}\ln(3) + \frac{1}{3}e^9 + \frac{3^6}{6} + C$$

$$C = 12 - \frac{1}{2}\ln(3) - \frac{1}{3}e^9 - \frac{3^6}{6}$$

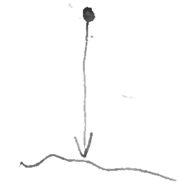
$$\approx -2811.08$$

Therefore

$$F(x) = \frac{1}{2}\ln(x) + \frac{1}{3}e^{3x} + \frac{x^6}{6} - 2811.08$$

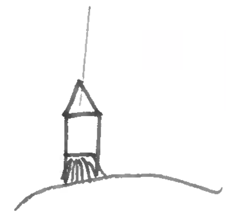
FACTS: * gravity on Earth is $-32 \frac{ft}{s^2}$

acceleration $\quad -9.81 \frac{m}{s^2}$



* recall

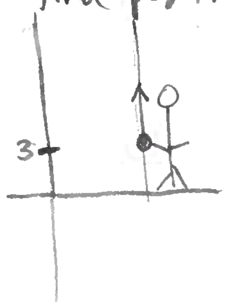
$$\text{position} \xrightarrow{\frac{d}{dx}} \text{velocity} \xrightarrow{\frac{d}{dx}} \text{acceleration}$$



$$\underset{s}{\text{position}} \xleftarrow{D^{-1} \; s'=v} \text{velocity} \xleftarrow{D^{-1} \; v'=a=s''} \text{acceleration}$$

Ex: If an object is thrown upwards with an initial velocity of $v(0) = 60 \frac{ft}{s}$ from initial position $s(0)=3$ then find position function for the object. (Assume only gravity affects the ball.)

Soln:



$-16t^2 + v_0 t + x_0$

$$a(t) = -32$$

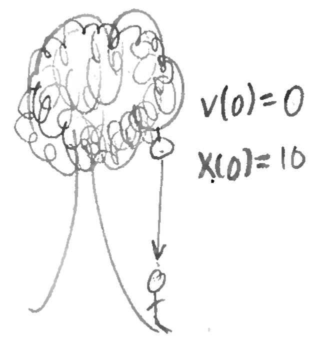
$$v(t) = D^{-1} a(t) = D^{-1}(-32)$$
$$= -32t + C$$

$$s(t) = D^{-1} v(t) = D^{-1}(-32t + C)$$
$$= -\frac{32t^2}{2} + Ct + D$$
$$= -16t^2 + Ct + D$$

Now apply initial conditions:

$$\begin{cases} 60 \underset{\substack{\uparrow \\ \text{given}}}{=} v(0) \underset{\substack{\uparrow \\ \text{computed}}}{=} -32(0)+C \\[2em] 3 \underset{\substack{\uparrow \\ \text{given}}}{=} s(0) \underset{\substack{\uparrow \\ \text{compute}}}{=} -16(0^2)+C(0)+D \end{cases}$$

$$\begin{cases} 60 = C \\ 3 = D \end{cases}$$



$\Rightarrow$ height of object at time t is

$$\boxed{s(t) = -16t^2 + 60t + 3}$$

Question: How long did it take for the object to hit the ground?

Need to solve $\underbrace{s(t)}_{\text{height}} = 0$:

$$-16t^2 + 60t + 3 = 0$$

$$t = \frac{-60 \pm \sqrt{60^2 - 4(-16)(3)}}{(-32)}$$

$$= \frac{60}{32} \pm \left(\frac{1}{-32}\right)\sqrt{} \approx$$

$\oplus$ $\ominus$

$\cancel{t = -0.049}$ Nonsensical soln $t = 3.84$

Question: What was the max height of the object?

$$s'(t) = -32t + 60 \overset{\text{set}}{=} 0$$

$$t = \frac{-60}{-32} = \frac{60}{32} = \frac{30}{16} = \frac{15}{8}$$

$$s''(t) = -32 < 0 \Rightarrow \text{by 2}^{\text{nd}} \text{ deriv test}$$

the max occurs

at $t = \frac{15}{8} = 1.875$

& its value is

$$s\left(\frac{15}{8}\right) = \frac{237}{4} = 59.25\text{ ft}$$