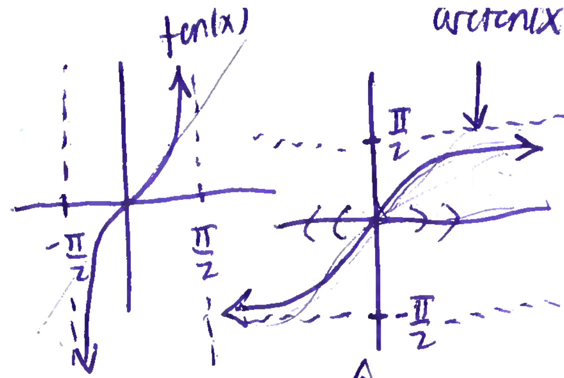
Ex: $\lim\limits_{x\to\infty} \arctan(x^2 - x^4)$



To answer this, just determine if

$x^2 << x^4$

$$\lim_{x\to\infty} (x^2 - x^4) = \lim_{x\to\infty} \frac{x^2 - x^4}{1}\underbrace{\left(\frac{\frac{1}{x^4}}{\frac{1}{x^4}}\right)}_{=1}$$

$\infty - \infty$

expect $\stackrel{?}{=} -\infty$

~~try same trick as last time~~

$$= \lim_{x\to\infty} \frac{\overset{\nearrow 0}{\frac{1}{x^2}} - 1}{\underset{\searrow 0}{\frac{1}{x^4}}}$$

$$\stackrel{?}{=} -\frac{1}{0} = -\infty$$

NOT "indeterminant" ~ if you get $\frac{a}{0}$

$$\begin{bmatrix} a > 0 \to \infty \\ a < 0 \to -\infty \\ a = 0 \to \text{indeterm.} \end{bmatrix}$$

Observe:

$\lim\limits_{x\to\infty} \arctan(x) = \frac{\pi}{2}$

$\lim\limits_{x\to-\infty} \arctan(x) = -\frac{\pi}{2}$

Therefore,

$$\lim_{x\to\infty} \arctan(x^2 - x^4) = -\frac{\pi}{2}$$

Ex: $\lim_{x\to\infty} x^2 e^{-x}$ $\leftarrow$ $e^{-x} = \frac{1}{e^x}$

$$= \lim_{x\to\infty} \underbrace{\frac{x^2}{e^x}}_{\frac{\infty}{\infty}} \overset{LH}{=} \lim_{x\to\infty} \underbrace{\frac{2x}{e^x}}_{\frac{\infty}{\infty}} \overset{LH}{=} \lim_{x\to\infty} \frac{2}{e^x} = 0$$

Ex: $$\lim_{x\to 0} \frac{\sin x}{x} \overset{\frac{0}{0}}{\underset{LH}{=}} \lim_{x\to 0} \frac{\cos x}{1} = \cos(0) = 1$$

$$\lim_{x\to\infty} \frac{\sin(3x)}{x} \overset{LH}{=} \lim_{x\to\infty} \frac{3\cos(3x)}{1} = 3\cos(0) = 3$$

$\frac{0}{0}$

$1 \leftarrow 3\left(\frac{\sin(3x)}{3x}\right)$

$\ln(-1) = i\pi$

Ex: $$\lim_{x\to 0^+} (5x)^{x} = \lim_{x\to 0^+} e^{\ln((5x)^x)}$$

0^0

$$= \cancel{\lim e} = \lim_{x\to 0^+} e^{\boxed{x\ln(5x)}}$$

$2^x = e^{\ln(2^x)} = e^{x\ln(2)}$

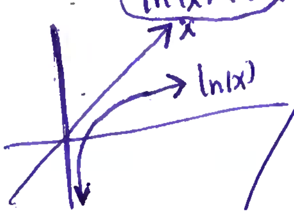
Look at $$\lim_{x\to 0^+} x\ln(5x) = \lim_{x\to 0^+} \frac{\ln(5x)}{\frac{1}{x}} \overset{LH}{=} \lim_{x\to 0^+} \frac{\frac{5}{5x}}{-\frac{1}{x^2}}$$

$0\cdot(-\infty)$ $\quad \frac{-\infty}{\infty}$

$$= \lim_{x\to 0^+} -\frac{5x^2}{5x}$$

$$= \lim_{x\to 0^+} -x = 0$$

$\boxed{\ln(x) << x}$



Therefore
$$\lim_{x\to 0^+} (5x)^x = e^0 = 1$$

Ex: $\lim_{x\to 0} (1+7x)^{3/x} = \lim_{x\to 0} e^{\ln\left((1+7x)^{3/x}\right)}$

1^{∞}

$= \lim_{x\to 0} e^{\frac{3}{x}\ln(1+7x)}$

We look at

$$\boxed{\lim_{x\to 0} \frac{3\ln(1+7x)}{x} \overset{LH}{=} \lim_{x\to 0} \frac{\frac{3}{1+7x}(7)}{1} = 21}$$

$x=0$

$\frac{3\ln(1)}{0} = \frac{0}{0}$

So,

$\lim_{x\to 0} (1+7x)^{\frac{3}{x}} = e^{21}$

Antiderivatives

Idea: find a function whose derivative is the given function

Ex: What are some functions whose derivative is 1?

$$x$$

$$x+2$$

in general: $x + C$ ⟵ all antiderivatives of 1 are of this form

Earlier: notation for derivatives:

$$D^1 f = \frac{df}{dx} = f'(x)$$

$$D^2 f = \frac{d^2 f}{dx^2} = f''(x)$$

Define:

$$\boxed{D^{-1} f = \text{antiderivatives of } f}$$

Ex: $D^{-1}x$

Soln: $\frac{x^2}{2} + C$

check

$\frac{d}{dx}\left(\frac{x^2}{2}\right) = \frac{1}{2}\frac{d}{dx}x^2$

$= \frac{1}{2}(2x)$

$= x$ ✓

Ex: $D^{-1}x^2$

Soln: $\frac{x^3}{3} + C$

$\frac{d}{dx}x^4 = 4x^3$

$\frac{d}{dx}x^3 = 3x^2$

$\frac{d}{dx}\frac{x^3}{3} = x^2$

Ex: $D^{-1}x^3$

Soln: $\frac{x^4}{4} + C$

General formula:

$$D^{-1}x^n = \frac{x^{n+1}}{n+1} + C$$

(compare to $Dx^{n+1} = (n+1)x^n$)