$$\frac{d}{dx}e^{x} = e^{x} \qquad \frac{d}{dx}\sin(x) = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

match

$$(fg)' = f'g + fg'$$

$$\boxed{\text{new: } \frac{d}{dx}\ln(x) = \frac{1}{x}}$$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

Ex: $p(\alpha) = \ln(\alpha) + e^{\alpha}\cos(\alpha)$

$$p'(\alpha) = \frac{d}{d\alpha}[\ln(\alpha)] + \frac{d}{d\alpha}[e^{\alpha}\cos(\alpha)]$$

$$= \frac{1}{\alpha} + \left[\frac{d}{d\alpha}[e^{\alpha}]\cos(\alpha) + e^{\alpha}\frac{d}{d\alpha}[\cos(\alpha)]\right]$$

$$= \frac{1}{\alpha} + e^{\alpha}\cos(\alpha) + e^{\alpha}(-\sin(\alpha))$$

~~Ex:~~ $\cancel{\frac{d}{dx}(3^{x})}$

Ex: If $y = \frac{x}{2x+1}$, find $\left.\frac{dy}{dx}\right|_{x=3}$

take derivative AND THEN set $x=3$

Soln: Quotient Rule:

$$\frac{dy}{dx} = \frac{(2x+1)\frac{d}{dx}[x] - x\frac{d}{dx}[2x+1]}{(2x+1)^2}$$

$$= \frac{(2x+1)(1) - x(2)}{(2x+1)^2}$$

$$= \frac{1}{(2x+1)^2} \Longrightarrow \left.\frac{dy}{dx}\right|_{x=3} = \frac{1}{(2\cdot 3+1)^2} = \frac{1}{49}$$

$f'(a)$ $\frac{df}{dx}\Big|_{x=a}$ $g(x) = x^2$

$g'(a) = 2a$

$g'(☺) = 2☺$

Ex: Differentiate $\frac{\csc(x)}{e^x}$

$(e^a)^b = e^{ab}$

Soln: $$\frac{d}{dx}\left[\frac{\csc(x)}{e^x}\right] = \frac{e^x \frac{d}{dx}[\csc(x)] - \csc(x)\frac{d}{dx}[e^x]}{(e^x)^2}$$

$$= \frac{-e^x\cot(x)\csc(x) - e^x\csc(x)}{e^{2x}}$$

$$\frac{d}{dx}\csc(x) = \frac{d}{dx}\left[\frac{1}{\sin(x)}\right]$$

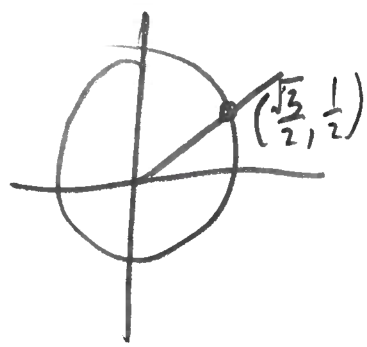
$$= \frac{0 - \cos(x)}{\sin^2 x}$$

$$= -\cot(x)\csc(x)$$

Ex: Find eqt of tan line to

$$W(\theta) = \ln(\theta) + \sin(\theta)\cos(\theta)$$

at $\theta = \frac{\pi}{6}$.



Soln: $W\left(\frac{\pi}{6}\right) = \ln\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{6}\right)\cos(\pi/6)$

$= \ln\left(\frac{\pi}{6}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \Longrightarrow \left(\frac{\pi}{6}, \ln\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{4}\right)$

$$W'(\theta) = \frac{d}{d\theta}[\ln(\theta)] + \frac{d}{d\theta}[\sin(\theta)\cos(\theta)]$$

$$= \frac{1}{\theta} + [\cos^2(\theta) - \sin^2(\theta)]$$

$$W'\left(\frac{\pi}{6}\right) = \frac{1}{\pi/6} + \cos^2\left(\frac{\pi}{6}\right) - \sin^2(\pi/6) = \frac{6}{\pi} + \frac{3}{4} - \frac{1}{4} = \frac{6}{\pi} + \frac{1}{2}$$

tan line: $y - \left(\ln\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{4}\right) = \left(\frac{6}{\pi} - \frac{1}{2}\right)\left(x - \frac{\pi}{6}\right)$

Chain Rule

* function composition

$f(x) = \sin(x) \quad g(x) = x^2+3$

$f(g(x)) = f(x^2+3)$
$= \sin(x^2+3)$

cannot do with methods we know!

$g(f(x)) = g(\sin(x))$
$= \sin^2(x)+3$

could do w/ product Rule

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx} \sin(x^2+3)$$

do not match! $\Rightarrow$ Cannot use $\frac{d}{dx}\sin(x) = \cos(x)$ here!!

We can say

$$\frac{d}{d(x^2+3)} \sin(x^2+3) = \cos(x^2+3)$$

seems weird — but it works!

$\frac{0}{0}$

Abuse of notation

Generally – "$\frac{dy}{dx}$" is NOT a fraction

BUT it is ~~often~~ convenient sometimes to pretend that it is!

$\frac{d}{dx}x = 1$

$a^2+b^2=c^2$

$\frac{d}{dx}\sin(x^2+3)$ we know $\frac{d}{d(x^2+3)}\sin(x^2+3) = \cos(x^2+3)$

TRICK: "$\frac{d}{dx} = \underbrace{\left(\frac{d(x^2+3)}{d(x^2+3)}\right)}_{=1}\frac{d}{dx} = \underbrace{\frac{d(x^2+3)}{dx}}\frac{d}{d(x^2+3)}$"

SWAP

In this case:

$$\frac{d}{dx}\sin(x^2+3) = \left(\frac{d}{dx}[x^2+3]\right)\frac{d}{d(x^2+3)}\sin(x^2+3)$$

$$= 2x\cos(x^2+3)$$

Other approach:

$$\frac{d}{dx}f(g(x)) = \underbrace{f'(g(x))}_{\text{"derivative of outside"}}\underbrace{g'(x)}_{\text{"derivative of inside"}}$$

mismatch!

Ex: $\dfrac{d}{dx} e^{-x^2} = \dfrac{d}{dx}\left[-x^2\right] \dfrac{d}{d(-x^2)} e^{-x^2}$

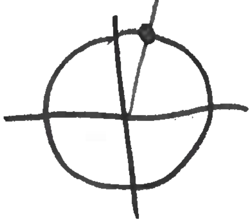
$= -2x e^{-x^2}$

$\dfrac{d}{dx} e^{x} = e^{x}$

$\dfrac{d}{dx} = \dfrac{d(-x^2)}{dx} \dfrac{d}{d(-x^2)}$

Ex: Find tan line to $T(\Omega) = \sin(\cos(\Omega))$ at $\Omega = \frac{\pi}{3}$.

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



Soln: pt ~ $T\left(\frac{\pi}{3}\right) = \sin\left(\cos\left(\pi/3\right)\right) = \sin\left(\frac{1}{2}\right)$

$\Rightarrow \left(\frac{\pi}{3}, \sin\left(\frac{1}{2}\right)\right)$

$T'(\Omega) = \dfrac{d}{d\Omega} \sin(\cos(\Omega))$

mismatch

$\dfrac{d}{d\alpha} \sin\alpha = \cos\alpha$

$= \dfrac{d(\cos(\Omega))}{d\Omega} \dfrac{d}{d(\cos(\Omega))} \sin(\cos\Omega)$

$\dfrac{d}{d\alpha} \cos\alpha = -\sin\alpha$

$= -\sin(\Omega)\cos(\cos(\Omega))$

$T'(\pi/3) = -\sin(\pi/3)\cos(\cos(\pi/3)) = -\frac{\sqrt{3}}{2}\cos\left(\frac{1}{2}\right)$

tan line: $y - \sin\left(\frac{1}{2}\right) = -\frac{\sqrt{3}}{2}\cos\left(\frac{1}{2}\right)\left(x - \frac{\pi}{3}\right)$