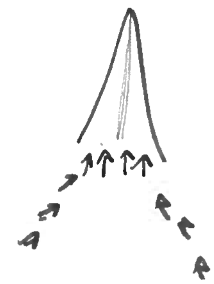
Units

→ $f(t)$ ~ measured in foo

t ~ measured in bar



$\Downarrow$

→ $f'(t)$ ~ measured in $\dfrac{\text{foo}}{\text{bar}}$

$$f'(t) = \lim_{h\to 0} \frac{f(\cancel{t}+h) - f(t)}{h}$$

Recall: $\dfrac{d}{dx}\, x^n = n x^{n-1}$

PR Power Rule

"take the derivative of"

$$\frac{d}{dx}\big(f(x) + g(x)\big) = f'(x) + g'(x)$$

SR Sum Rule

"derivative of sum is sum of derivatives"

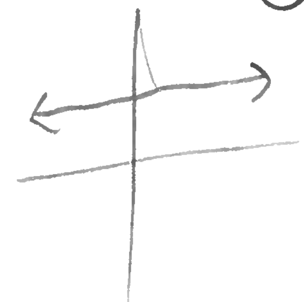
$$\frac{d}{dx}\big(c f(x)\big) = c f'(x)$$

constant

CMR Const. Mult. Rule

"the deriv. of a constant times f is that constant times deriv. of f"

Ex: Calculate $\quad x = x^1 \quad 1 = x^0$



$$\frac{d}{dx}\left[3x^2-2x+1\right]$$

SR

$$= \frac{d}{dx}\left[3x^2\right] + \frac{d}{dx}\left[-2x\right] + \frac{d}{dx}\left[1\right]$$

CMR

$$= 3\frac{d}{dx}\left[x^2\right] - 2\frac{d}{dx}\left[x^1\right] + \frac{d}{dx}\left[x^0\right] \rightarrow 0$$

PR

$$= 3\cdot 2x^{2-1} - 2\cdot 1\cdot x^{1-1} + \cancel{0x^{0-1}}$$

$$= 6x-2 \leftarrow f'(x)$$

Find eqt of tan line to $f(x) = 3x^2-2x+1$ at $x=3$.

$$f(3) = 3(3^2)-2(3)+1 = 3(9)-6+1$$
$$= 27-6+1$$
$$= 22$$

$\checkmark \rightarrow (3, 22)$

Slope: $f'(3) = 6(3)-2 = 18-2 = 16$

$$\begin{array}{r} 16 \\ 3 \\ \hline 48 \end{array}$$

$$y-y_0 = m(x-x_0)$$
$$y-22 = 16(x-3)$$
$$y = 16x-48+22 = 16x-26$$

Ex: Find eqt of tan line to

$$f(x) = \frac{1}{2}x^8 - x^7 + 2x^6 - 49$$

at $x = -1$.

Soln:

$$\left.\begin{aligned} f(-1) &= \tfrac{1}{2}(-1)^8 - (-1)^7 + 2(-1)^6 - 49 \\ &= \tfrac{1}{2} + 1 + 2 - 49 = -45.5 \end{aligned}\right\} (-1, -45.5)$$

$(-1)^2 = 1$

$(-1)^3 = -1$

$\vdots$

$(-1)^{\text{even}} = 1$

$(-1)^{\text{odd}} = -1$

$$f'(x) = \frac{d}{dx}\left[\frac{1}{2}x^8 - x^7 + 2x^6 - 49\right]$$

$$\overset{SR}{=} \frac{d}{dx}\left[\frac{1}{2}x^8\right] + \frac{d}{dx}\left[-x^7\right] + \frac{d}{dx}\left[2x^6\right] + \frac{d}{dx}\left[-49\right]$$

$$\overset{CMR}{=} \frac{1}{2}\frac{d}{dx}\left[x^8\right] - \frac{d}{dx}\left[x^7\right] + 2\frac{d}{dx}\left[x^6\right] - 49\frac{d}{dx}\left[1\right]$$

$$\overset{PR}{=} \frac{8}{2}x^7 - 7x^6 + 12x^5 - 0$$

slope at $x = -1$:

$$\begin{aligned} f'(-1) &= 4(-1)^7 - 7(-1)^6 + 12(-1)^5 \\ &= -4 - 7 - 12 \\ &= -23 \end{aligned}$$

eqt of tan line: $y - (-45.5) = -23(x - (-1))$

Ex: $f(t) = t^{\pi}$ $\qquad \pi \approx 3.14\ldots$

$f'(t) = \pi t^{\pi - 1}$

Ex: $\frac{d}{d\xi}\left(2\xi^3 + \xi\right) = 6\xi + 1$

$\frac{d}{dx}\left(2x^3 + x\right) = 6x + 1$

Ex: $\frac{d}{d\,\text{☺}}\left[\text{☺}^4 - 3\text{☺}^2 + 1\right] = 4\text{☺}^3 - 6\text{☺}$

$\frac{d}{d\Omega}\left[\psi^3 + 2\psi - 1\right]$???

mismatch

$\Downarrow$

cannot use power Rule

<u>EX</u>: $f(x) = \frac{1}{x} + \frac{1}{x^2}$

<u>Recall</u>: $\frac{1}{x} \equiv x^{-1}$ generally: $\frac{1}{x^m} \equiv x^{-m}$

$\frac{1}{x^2} \equiv x^{-2}$

[<u>Advice</u>: rewrite all $\frac{1}{x^m}$ as x^{-m}]

$$f'(x) = \frac{d}{dx}\left[\frac{1}{x} + \frac{1}{x^2}\right]$$

$$\overset{SR}{=} \frac{d}{dx}\left[x^{-1}\right] + \frac{d}{dx}\left[x^{-2}\right]$$

$$\overset{PR}{=} (-1)x^{-1-1} + (-2)x^{-2-1}$$

$$= -x^{-2} - 2x^{-3}$$

$$= -\frac{1}{x^2} - \frac{2}{x^3}$$

Eqt of tan line to f at $x = 2$

$$f(2) = \frac{1}{2} + \frac{1}{2^2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \Longrightarrow \left(2, \frac{3}{4}\right)$$

$$f'(2) = -\frac{1}{2^2} - \frac{2}{2^3} = -\frac{1}{4} - \frac{2}{8} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$$

$$y - \frac{3}{4} = -\frac{1}{2}(x - 2)$$

$$x^a x^b = x^{a+b}$$

<u>Recall</u>: $\sqrt{x} \equiv x^{1/2}$ $\qquad (\sqrt{x})^2 = x$

$$x^{1/2} \cdot x^{1/2} = x^{1/2+1/2} = x^1$$

$$\sqrt[3]{x} \equiv x^{1/3}$$

$$\vdots$$

$$\sqrt[n]{x^m} = x^{\frac{m}{n}} \qquad \frac{1}{\sqrt{x}} \equiv x^{-1/2}$$

<u>Ex</u>: Find eqt of tan line to

$$f(x) = \sqrt[3]{x} + \sqrt{x} - \frac{1}{x}$$

at $x=3$

<u>Soln</u>: $f(3) = \sqrt[3]{3} + \sqrt{3} - \frac{1}{3} \longrightarrow \left(3, 3^{1/3} + 3^{1/2} - \frac{1}{3}\right)$

$$f'(x) = \frac{d}{dx}\left[x^{1/3} + x^{1/2} - x^{-1}\right]$$

$$\overset{SR}{=} \frac{d}{dx}\left[x^{1/3}\right] + \frac{d}{dx}\left[x^{1/2}\right] + \frac{d}{dx}\left[-x^{-1}\right]$$

$$\overset{PR+CMR}{=} \frac{1}{3}x^{1/3-1} + \frac{1}{2}x^{1/2-1} - (-1)x^{-1-1}$$

$$= \frac{1}{3}x^{-2/3} + \frac{1}{2}x^{-1/2} + x^{-2}$$

$$f'(3) = \frac{1}{3}3^{-2/3} + \frac{1}{2}3^{-1/2} + 3^{-2}$$

$$\Rightarrow y - \underbrace{\left(3^{1/3} + 3^{1/2} - \frac{1}{3}\right)}_{\text{a number}} = \underbrace{\left[\frac{1}{3}3^{-2/3} + \frac{1}{2}3^{-1/2} + 3^{-2}\right]}(x-3)$$