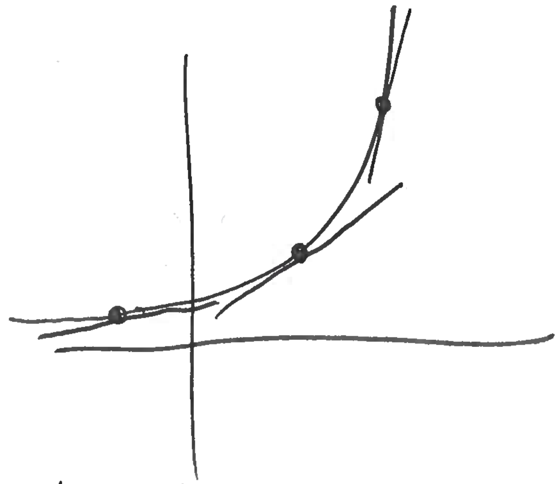
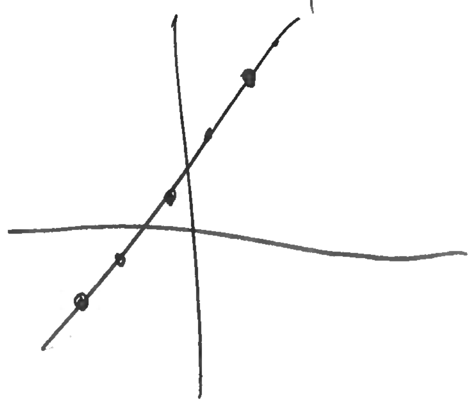
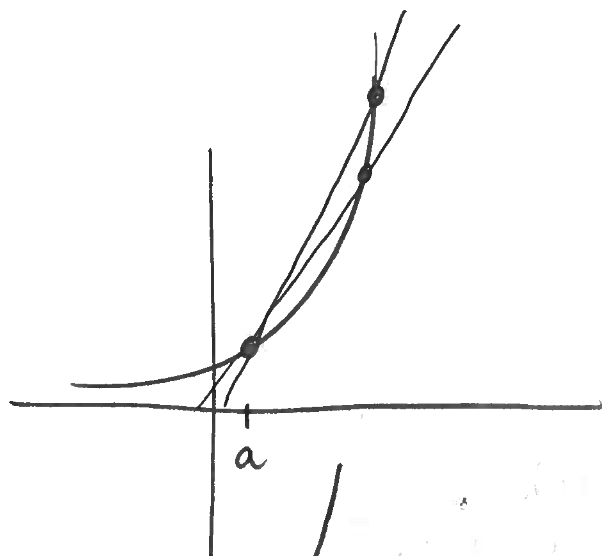
Tangent line problem

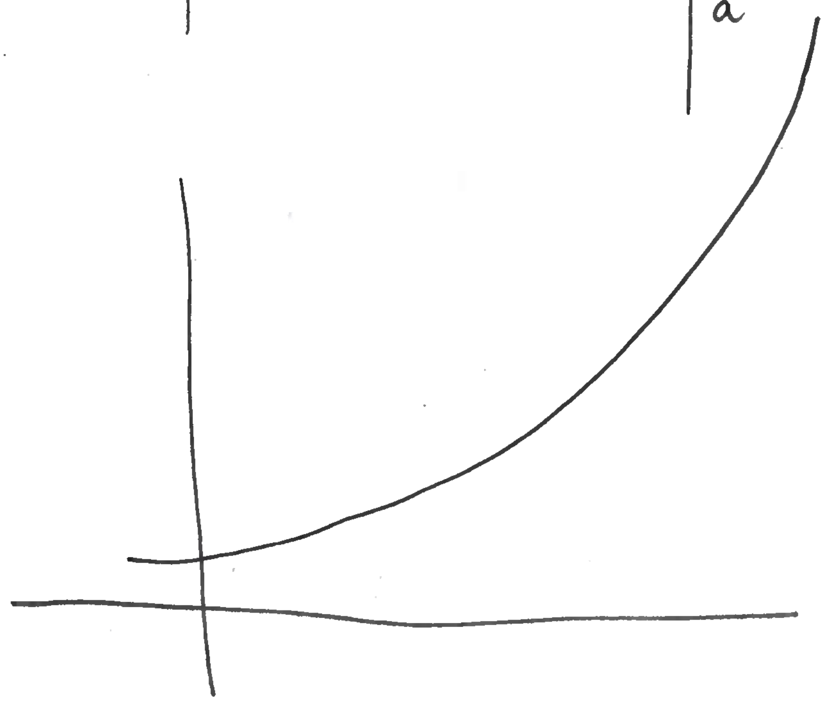
What is slope of a curve?

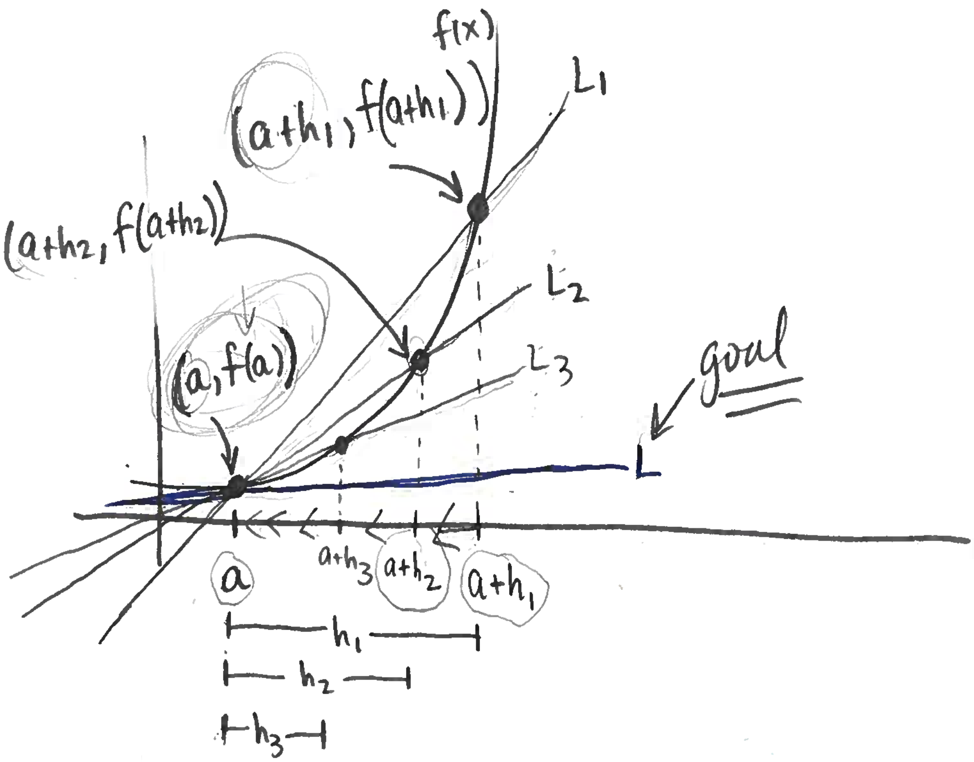
curve has different "instantaneous slope" at different point











* As the points get closer to a, their distance, h, from a gets smaller.
* When the points reach a, their distance, h, from a becomes zero.
* Slopes of L_1, L_2, L_3, etc get closer & closer to the slope of L.
* All I need to find tangent line L is:
  1. a point on L ~ it is $(a, f(a))$ ✓
  2. a slope for L ~ comes from a limit process

Find slope of L_1

$$\text{slope}_{L_1} = \frac{f(a+h_1) - f(a)}{(\cancel{a}+h_1) - \cancel{a}} = \frac{f(a+h_1) - f(a)}{h_1}$$

Slope of L_2

$$\text{slope}_{L_2} = \frac{f(a+h_2) - f(a)}{(\cancel{a}+h_2) - \cancel{a}} = \frac{f(a+h_2) - f(a)}{h_2}$$

Definition of slope of tangent line:

$$[\text{Slope of } f \text{ at } (x=a)] \overset{\text{def}}{=} \underbrace{\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}}_{\text{this will be a number called "the derivative of } f \text{ at } x=a\text{"}}$$

Ex: Find instanteneous slope of $f(x) = x^2$ at $x=2$.
Use this to find eqt of tangent line there.
Do same at $x=3$.

Soln: $f(2) = 2^2 = 4 \rightsquigarrow (2,4)$

$$[\text{Slope of } f \text{ at } (x=2)] = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$(2+h)^2 = (2+h)(2+h)$
$= 4+2h+2h+h^2$
$= 4+4h+h^2$

$$= \lim_{h\to 0} \frac{(2+h)^2 - 2^2}{h}$$

$$= \lim_{h\to 0} \frac{[\cancel{4}+4h+h^2] - \cancel{4}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4+h)}{\cancel{h}} = \lim_{h\to 0} (4+h) = 4+0 = 4$$

Now the tangent line is line thru $(2,4)$ with slope $m=4$.

Recall: point-slope form of eqt of line ~ slope m, point (x_0,y_0) on line

$$y-y_0=m(x-x_0)$$

Taking $m=4$, $(x_0,y_0)=(2,4)$

$$y-4=4(x-2)$$

$$y=4x-8+4$$

$$\boxed{y=4x-4}$$

Do same at $x=3$: $f(x)=x^2$

$$f(3)=9 \longrightarrow (3,9)$$

$$[\text{slope of } f \text{ at } (x=3)]=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$$

$$=\lim_{h\to 0}\frac{(3+h)^2-9}{h}$$

$$=\lim_{h\to 0}\frac{(\cancel{9}+6h+h^2)-\cancel{9}}{h}$$

$$=\lim_{h\to 0} 6+h = 6$$

$m=6$, $(x_0,y_0)=(3,9)$

$$y-9=6(x-3) \longrightarrow y=6x-18+9=6x-9$$

Ex: Again for $f(x)=x^2$, find

$$[\text{slope of } f \text{ at } x=x] = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$$

$$= \lim_{h\to 0} \frac{(\cancel{x^2}+2x\cancel{h}+h^{\cancel{2}})-\cancel{x^2}}{\cancel{h}}$$

$$= \lim_{h\to 0} 2x+h = 2x$$

the derivative function

$f'(x)$ or $\frac{df}{dx}$ or Df or $\dot{f}$

From this, we can find tangent lines very quickly: $\boxed{f'(x)=2x}$

at $(-5,25)$: slope at this point is

$$f'(-5) = 2(-5) = -10$$

$$\Rightarrow y-25 = -10(x-(-5))$$

$$y = -10x-50+25$$

$$= -10x-25$$

$$\begin{array}{r} 0.4 \\ 0.4 \\ \hline .16 \end{array}$$

at $(0,0)$:

$$f'(0) = 2(0) = 0$$

$$\Rightarrow y-0 = 0(x-0)$$

$$y = 0$$

at $(0.4, 0.16)$

$$f'(0.4) = 2(0.4) = 0.8$$

$$\Rightarrow y-0.16 = 0.8(x-0.4)$$

$$y = 0.8x-0.32+0.16$$

$$= 0.8x-0.16$$