FRIDAY

Będlewo

Theorem: $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$

"If it rains, then I will bring an umbrella"

P

Q

$P \to Q$

different

"If I bring an umbrella, then it rains."

$Q \to P$

Properties of limits

$$\lim_{x\to a}\left[f(x) \pm g(x)\right] = \left[\lim_{x\to a} f(x)\right] \pm \left[\lim_{x\to a} g(x)\right]$$

$$\lim_{x\to a}\left[f(x)\cdot g(x)\right] = \left[\lim_{x\to a} f(x)\right]\left[\lim_{x\to a} g(x)\right]$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

$$\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$$

constant

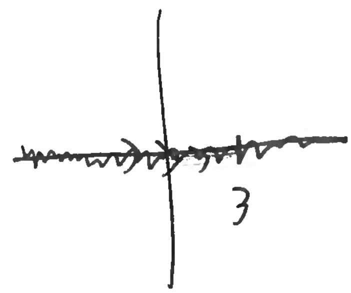
★ ~ provided that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist

Ex: THIS DOESN'T WORK:

$$0 = \lim_{x\to 3} 0 = \lim_{x\to 3}\left(\frac{1}{x-3} - \frac{1}{x-3}\right)$$

BAD

$$= \left(\lim_{x\to 3}\frac{1}{x-3}\right) - \left(\lim_{x\to 3}\frac{1}{x-3}\right)$$



$\frac{1}{0}$ DONT EXIST

Ex:

$$\lim_{x\to 2}\left(x^2+3x-1\right) \overset{(x=2)}{=} 2^2+3(2)-1 = 4+6-1 = 9$$

$$\| $$

$$\left(\lim_{x\to 2} x^2\right) + 3\left(\lim_{x\to 2} x\right) - \left(\lim_{x\to 2} 1\right)$$

$$\| $$

$$2^2+3(2)-1$$

$$\| $$

$$9$$

Ex:

$$\lim_{x\to 2} \frac{\sqrt{x}+x^3}{\sin(x)-8\sqrt[3]{x}} \overset{(x=2)}{=} \frac{\sqrt{2}+8}{\sin(2)-8\sqrt[3]{2}}$$

EX: $\displaystyle\lim_{x\to 2} \frac{x^2-x-2}{x^2+x-6} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+1)}{(x+3)\cancel{(x-2)}}$

$\boxed{x=2}$

$\dfrac{2^2-2-2}{2^2+2-6} = \dfrac{4-4}{6-6} = \dfrac{0}{0} \to$ MORE WORK TO DO

$\displaystyle = \lim_{x\to 2} \frac{x+1}{x+3}$

$= \dfrac{2+1}{2+3} = \dfrac{3}{5}$

EX: $\displaystyle\lim_{x\to -1} \frac{x+1}{x^2+x} = \lim_{x\to -1} \frac{\cancel{(x+1)}}{x\cancel{(x+1)}}$

$x=-1$

$-1^2=-1$

$(-1)^2=1$

$\dfrac{-1+1}{(-1)^2+(-1)} = \dfrac{0}{1-1} = \dfrac{0}{0}$

MORE WORK

$\displaystyle = \lim_{x\to -1} \frac{1}{x}$

$= -1$

EX: Given $\displaystyle\lim_{x\to 2} f(x) = 10$ and $\displaystyle\lim_{x\to 2} g(x) = 8$

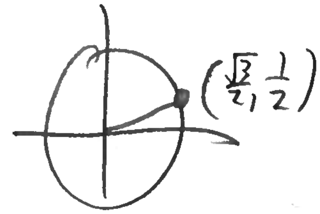
What is $\displaystyle\lim_{x\to 2} \frac{f(x)}{g(x)}$?

$$\lim_{x\to 2} \frac{f(x)}{g(x)} = \frac{\displaystyle\lim_{x\to 2} f(x)}{\displaystyle\lim_{x\to 2} g(x)} = \frac{10}{8}$$

EX: Recall: $\boxed{\lim_{x\to 0}\frac{\sin x}{x} = 1}$

$x=0$

$\frac{\sin(0)}{0} = \frac{0}{0}$



$$\lim_{x\to\frac{\pi}{6}}\frac{\sin x}{x} = \frac{\sin(\pi/6)}{\pi/6} = \frac{1/2}{\pi/6} = \left(\frac{1}{2}\right)\left(\frac{6}{\pi}\right) = \frac{3}{\pi}$$

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{a}{b}\cdot\frac{d}{c}$$

$$\lim_{w\to 0}\frac{\tan(8w)}{\sin(2w)} = \lim_{w\to 0}\frac{\sin(8w)}{\sin(2w)\cos(8w)}$$

$$\boxed{\tan(8w) = \frac{\sin(8w)}{\cos(8w)}}$$

$$= \lim_{w\to 0}\left[\frac{\sin(8w)}{\sin(2w)\cos(8w)}\cdot\frac{(8w)(2w)}{(8w)(2w)}\right]$$

$w=0$

$$\frac{\sin(0)}{\sin(0)\cos(0)} = \frac{0}{0\cdot 1} = \frac{0}{0} \to \text{more work}$$

$$\frac{\frac{\sin(8w)}{\cos(8w)}}{\frac{\sin(2w)}{1}}$$

$$= \lim_{w\to 0}\underbrace{\left(\frac{\sin(8w)}{8w}\right)}_{1}\underbrace{\left(\frac{2w}{\sin(2w)}\right)}_{1}\left(\frac{8w}{(2w)\cos(8w)}\right)$$

$$1 = \frac{8w}{8w}\cdot\frac{2w}{2w}$$

$$= (1)\cdot(1)\left(\lim_{w\to 0}\frac{4}{\cos(8w)}\right)$$

$w=0$

$$= \frac{4}{\cos(0)} = \frac{4}{1} = 4$$

Remember:

$\tan x = \frac{\sin x}{\cos x}$

$\sec x = \frac{1}{\cos x}$ $\qquad \cot x = \frac{\cos x}{\sin x}$

$\csc x = \frac{1}{\sin x}$