$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

Ex: Evaluate

$$\log_7\left(\sqrt[4]{7}\right) = \cancel{\log_7}\left(\cancel{7}^{1/4}\right)$$

$$= \frac{1}{4}$$

Recall

$\log_7(x)$ is inverse of

7^x

↓ means

$$\boxed{\begin{array}{c} 7^{\log_7(x)} = x \\ \log_7(\cancel{7}^x) = x \end{array}}$$

Ex: Evaluate

$$\log_2(64) = \cancel{\log_2}(\cancel{2}^6) = 6$$

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64

can I write 64 has 2^n for some n?

Yes ~ $64 = 2^6$

Ex: Evaluate $\log_4(64)$

$$64 = 4^3$$

n	4^n
0	1
1	4
2	16
3	64

So,

$$\log_4(64) = \cancel{\log_4}(\cancel{4}^3) = 3$$

Ex: $\log_3(12) = \log_3(3\cdot 4) = \log_3(3^1) + \log_3(4)$

Can 12 be written as 3^n?

NO

$= 1 + \underbrace{\log_3(4)}_{\text{does not simplify}}$

Ex: Compute $\ln(e^5)$

Recall: "ln" means "$\log_e$" ← special #

$\cancel{\ln}(\cancel{e}^5) = 5$

inverse prop

Ex: $\cancel{2}^{\cancel{\log_2}(12)} = 12$

$f(x) = 2^x$

$f^{-1}(x) = \log_2(x)$

$x = f(f^{-1}(x)) = f(\log_2(x)) = 2^{\log_2(x)}$

$x = f^{-1}(f(x)) = f^{-1}(2^x) = \log_2(2^x)$

inverse property

Ex: Express as a single logarithm

$$3\log(x) - 5\log((x^2-1)) + 2\log((x+9))$$

$$= \underbrace{\log(x^3) - \log((x^2-1)^5)}_{\text{"difference of logs is log of quotient"}} + \log((x+9)^2)$$

$$= \underbrace{\log\left(\frac{x^3}{(x^2-1)^5}\right) + \log((x+9)^2)}_{\text{"sum of logs is log of product"}}$$

$$= \log\left(\frac{x^3(x+9)^2}{(x^2-1)^5}\right)$$

Ex: Expand as sums, differences, and multiples of simpler logarithms

$$\ln\left(\sqrt[8]{\frac{(x+1)^2(x-2)^5}{(x+3)^9}}\right) = \ln\left(\left(\frac{(x+1)^2(x-2)^5}{(x+3)^9}\right)^{1/8}\right)$$

$$= \frac{1}{8}\ln\left(\frac{(x+1)^2(x-2)^5}{(x+3)^9}\right)$$

$$= \frac{1}{8}\left[\ln((x+1)^2(x-2)^5) - \ln((x+3)^9)\right]$$

$$= \frac{1}{8}\left[\ln((x+1)^2) + \ln((x-2)^5) - 9\ln(x+3)\right]$$

$$= \frac{2}{8}\ln(x+1) + \frac{5}{8}\ln(x-2) - \frac{9}{8}\ln(x+3)$$

Ex: $\log_x(8) = 3$

Soln: Think: $8 = (?)^3 \leftarrow 8 = x^3$

$\downarrow$

$x = \sqrt[3]{8} = 2$

FACT: $8 = 2^3$

So,

$\log_2(8) = \log_2(2^3) = 3$ ✓

$x = 2$

Ex: $\log_x(16) = 2$ vs $\log_x(16) = 4$

$\uparrow$

write

$16 = x^2$

$\downarrow \sqrt{\ }$

$x = \pm 4$

$x = -4 \leadsto$ not for this class b/c it leads to $i = \sqrt{-1}$

$\Downarrow$

$\boxed{x = 4}$

$16 = 4^2$ ✓

$\uparrow$

write

$\boxed{16 = x^4} \leftarrow$ "quadratic in form"

$\downarrow$

let $w = x^2$

$\boxed{w^2 = (x^2)^2 = x^4}$

same as

$16 = w^2 \rightarrow w = \pm 4$

$\downarrow$

$\boxed{w = 4}$

Since $w = x^2$:

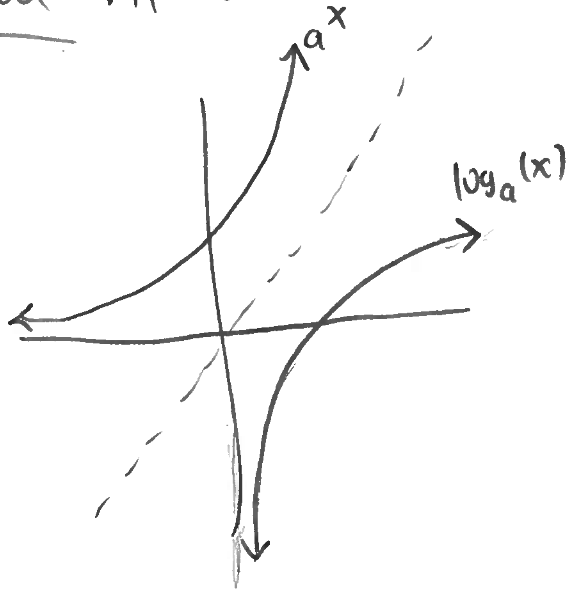
no negative

$x^2 = 4 \rightarrow x = \pm 2$

$\Rightarrow \boxed{x = 2}$

(makes sense: $16 = 2^4$)

Recall : if $a>1$



FACTS

$(x>0)$

$\mathrm{dom}(a^x) = \mathbb{R}$ $\qquad$ $\mathrm{dom}(\log_a(x)) = (0,\infty)$

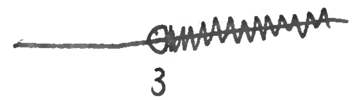
$\mathrm{range}(a^x) = (0,\infty)$ $\qquad$ $\mathrm{range}(\log_a(x)) = \mathbb{R}$

Ex: Find domain of $f(x) = \log(x-3)$

Soln: Require

$$x-3>0$$

$$x>3$$



$$\mathrm{dom}(f) = (3,\infty)$$

Ex: What is domain of

$$\log\left(x^2+x-2\right)?$$

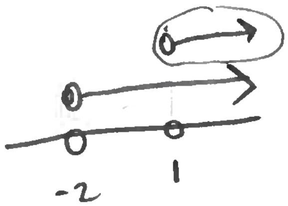
Soln: Require

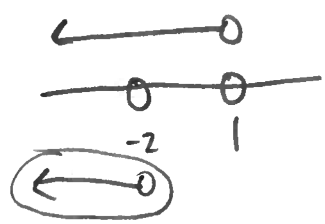
$$x^2+x-2>0$$

$$(x+2)(x-1)>0$$

OR

BOTH	BOTH
$x+2>0$	$x+2<0$
$x-1>0$	$x-1<0$
$\downarrow$	$\downarrow$
$x>-2$	$x<-2$
and	and
$x>1$	$x<1$





$$(-\infty,-2)\cup(1,\infty)$$