

Ex: Product of two consecutive even integers is 728. Find those integers.

Soln: Let  $x$  be one of the two integers.

The next consecutive even integer is  $x+2$ .

So,

$$x(x+2) = 728$$

$$x^2 + 2x - 728 = 0$$

$$\frac{a+b}{c} = \frac{a}{b} + \frac{b}{c} \quad \downarrow \text{QF}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-728)}}{2}$$

$$= \frac{-2 \pm \sqrt{4 + 2912}}{2}$$

$$= -1 \pm \frac{1}{2} \sqrt{2916}$$

$$= -1 \pm \frac{54}{2} = -1 \pm 27$$

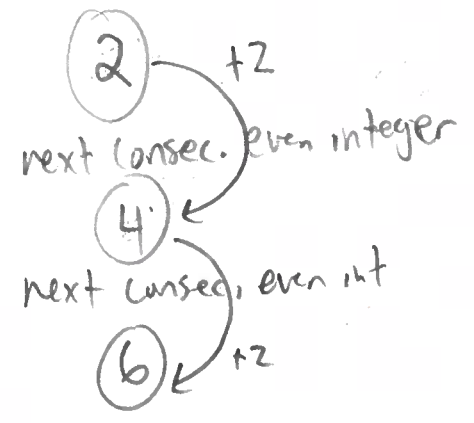
Use  $x=26 \rightarrow x+2=28$

Use  $x=-28 \rightarrow x+2=-26$

$\{-2, -1, 0, 1, 2, \dots\}$

$\{-2, 0, 2, 4, 6, \dots\}$

next to each other



$$ax^2 + bx + c = 0$$

# Rational equations and radical equations

(2)

means that we have fractions with polynomials (in  $x$ ) on top & on bottom

square roots  
cube roots, etc  
with  $x$  inside

## Radical eqts

Ex:  $\sqrt{x-1} = \sqrt{2x-3}$

recall  
 $(\sqrt{x})^2$  increases!

square both sides  
 $(\sqrt{x-1})^2 = (\sqrt{2x-3})^2$

inverse property  
 $x-1 = 2x-3$

$2 = x$

check  
 $\sqrt{2-1} \stackrel{?}{=} \sqrt{2(2)-3}$   
 $\sqrt{1} = \sqrt{1} \quad \checkmark$

BEWARE: when solving radical eqts, MUST ALWAYS check solns  $\sim$  i.e. plug back in to original

Ex:  $\sqrt{x} = x - 5$

square  
2

$x = (x - 5)^2$

$x = x^2 - 10x + 25$

$0 = x^2 - 11x + 25$

$121 - 100$

QF

$$x = \frac{-(-11) \pm \sqrt{121 - 4(1)(25)}}{2}$$

$$= \frac{11}{2} \pm \frac{1}{2} \sqrt{21}$$

⊕ / ⊖

$x = \frac{11}{2} + \frac{1}{2} \sqrt{21}$

$x = \frac{11}{2} - \frac{1}{2} \sqrt{21}$

Check

Check

$\sqrt{\frac{11}{2} + \frac{1}{2} \sqrt{21}} \stackrel{?}{=} \frac{11}{2} + \frac{1}{2} \sqrt{21} - 5$

$\sqrt{\frac{11}{2} - \frac{1}{2} \sqrt{21}} \stackrel{?}{=} \left(\frac{11}{2} - \frac{1}{2} \sqrt{21}\right) - 5$

$2.791 = 2.791 \checkmark$

$1.791 \stackrel{?}{=} -1.791$

NO

only soln

Ex:  $\sqrt{x} = \sqrt{-1-x}$

↓ square

$x = -1-x$

↓

$2x = -1$

↓

$x = -\frac{1}{2}$

$a, b > 0$   
 $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

check

$\sqrt{-\frac{1}{2}} = \sqrt{-1 - (-\frac{1}{2})}$

$\frac{1}{\sqrt{2}}i = \frac{1}{\sqrt{2}}i$  ✓

Ex:  $\sqrt{x-5} - \sqrt{x} = 1$

$\sqrt{x-5} = 1 + \sqrt{x}$

↓ square

$x-5 = (1+\sqrt{x})^2$

$x-5 = 1 + 2\sqrt{x} + x$

isolate + square again

$-5 = 1 + 2\sqrt{x}$

$-6 = 2\sqrt{x}$

$-3 = \sqrt{x}$

↓ square

$9 = x$

check

$\sqrt{9-5} - \sqrt{9} = 1$  ?

$\sqrt{4} - \sqrt{9} = 1$  ?

$2 - 3 = 1$  NO

NO SOLN