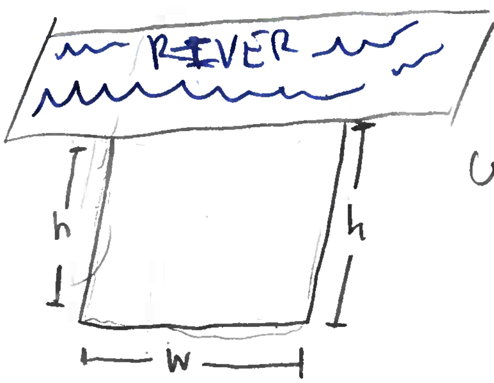
EX: You will build a rectangular sheep pen next to a river. No need to build along the river, so you need to only construct 3 sides.

You have a total of 350 feet of fencing to use.

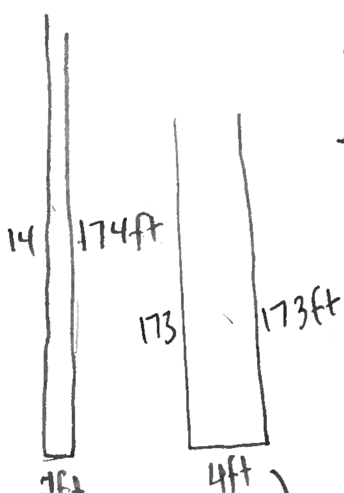
Find dimensions of a pen that encloses the max area.



constraint:

$h+w+h=350$

$\boxed{2h+w=350} \rightarrow \boxed{w=350-2h}$



$\boxed{\text{Area}=348\text{ft}^2}$

$\text{Area}=692\text{ft}^2$

MORE!!

Area:

$\boxed{\text{Area}_{pen} = wh}$

x^2+bx

$= \left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

Build constraint into area formula:

$\text{Area}_{pen} = (350-2h)h$

$= -2h^2+350h$

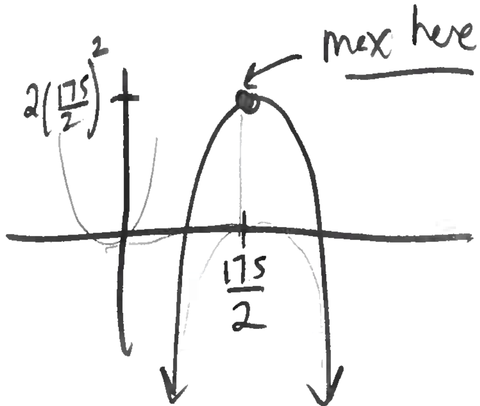
$= -2(\underbrace{h^2-175h}_{\text{complete square}}) = -2\left(\left(h-\frac{175}{2}\right)^2 - \left(\frac{175}{2}\right)^2\right)$

$= -2\left(h-\frac{175}{2}\right)^2 + 2\left(\frac{175}{2}\right)^2$

175
2)350
2
15
14

graph of h^2 shifted right by $\frac{175}{2}$, vert str refl by -2, w/ v. shift of $2\left(\frac{175}{2}\right)^2$

So, graph looks like



Observe:

Max area is $2\left(\frac{175}{2}\right)^2 \approx 15{,}312.5 \text{ ft}^2$

and occurs when $h = \frac{175}{2}$ and $w = 350 - 2\left(\frac{175}{2}\right)$

$$= 350 - 175$$
$$= 175$$

Ex: Two $\boxed{\text{numbers' sum is } -33}$ and $\boxed{\text{their product is } 17.6}$. Find the numbers.

Soln: Let 1st number be called x

2nd ______ y.

$$\begin{cases} x + y = -33 & \text{(i)} \\ xy = 17.6 & \text{(ii)} \end{cases}$$

From (i): $\boxed{x = -33 - y}$

$\downarrow$ plug into (ii)

$$(-33 - y)y = 17.6$$

$$-y^2 - 33y = 17.6 \longrightarrow 0 = y^2 + 33y + 17.6$$

QF $\uparrow$

$$y = \frac{-33 \pm \sqrt{33^2 - 4(1)(17.6)}}{2}$$

$+$: $y = -\frac{33}{2} + \frac{\sqrt{33^2 - 4(17.6)}}{2} \approx -0.542$

$-$: $y = -\frac{33}{2} - \frac{\sqrt{33^2 - 4(17.6)}}{2} \approx -32.457$

QF

if $ax^2 + bx + c = 0$,

then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So, we have

$y \approx -0.542$ OR $y \approx -32.457$

$$X = -33 - (-0.542)$$
$$= -32.458$$

it works!

$$X = -33 - (-32.457)$$
$$= -0.543$$

$$x + y = -33$$
$$x \cdot y = -17.59$$

rounds to -17.6

Ex: One number is 2 less than 3 times another number. Find a pair of such numbers so that their product is as small as possible.

$y = 1$
$x = 3(1) - 2 = 1$
$P = 1$

Soln: Let x and y be the numbers.

$$\longrightarrow X = 3y - 2$$

$y = 2$
$x = 3(2) - 2$
$= 4$
$P = 8$

Want to minimize $P = xy$

$y = 0$
$x = -2$
$P = 0$

$y = -1$
$x = 3(-1) - 2 = -5$
$P = 5$

$y = -1/2$
$x = 3(-1/2) - 2 = -3/2 - 2 = -5/2$
$P = \frac{5}{4}$

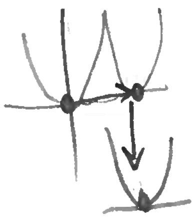
Plug $x=3y-2$ into $P=xy$:

$$P=(3y-2)y=3y^2-2y$$

$b=-2/3$

$$=3\left(y^2-\frac{2}{3}y\right)$$

$b=-\frac{2}{6}=\frac{-1}{3}$



complete square

$$=3\left(\left(y+1/3\right)^2-\left(\frac{1}{3}\right)^2\right)$$

graph of y^2 shifted right by $\frac{1}{3}$ + down by $\frac{1}{3}$ (v. str of 3)

$$=3\left(y-\frac{1}{3}\right)^2-\frac{1}{3}$$

$$\Downarrow$$

minimum at $y=\frac{1}{3}$ of $-\frac{1}{3}$

$$\downarrow$$

$$x=3\left(\frac{1}{3}\right)-2$$
$$=-1$$
$$P=-\frac{1}{3}$$