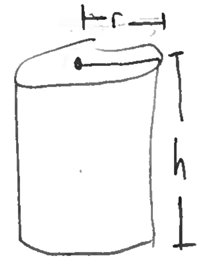
Ex: The volume of a right circular cylinder is

$\pi \approx 3.141$

$$V = \pi r^2 h$$

radius

height



What is the radius of such a cylinder if it has $\boxed{\text{height } 3}$ and volume 12π?

Soln:

$$12 = \pi r^2 (3)$$

solve for r

divide by 3π

$$r^2 = \frac{12}{3\pi} = \frac{4}{\pi}$$

$\sqrt{\ }$

We got two answers:

$$r = \pm\sqrt{\frac{4}{\pi}}$$

$r = \sqrt{\frac{4}{\pi}}$

$r = -\sqrt{\frac{4}{\pi}}$ not physically meaningful ← negative length doesn't make sense!

So,

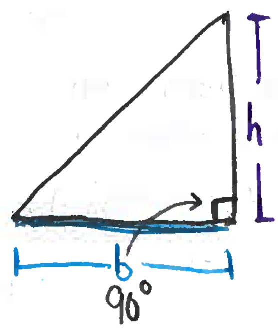
$$r = \sqrt{\frac{4}{\pi}} \approx 1.13$$

Ex: height of a right triangle is 10 feet more than 3 times the length of its base $\leftarrow h$

Express area as a function of the length of its base, x, in feet.

$+10$ $3x$ $b = x$

Soln:



$$\text{Area}_\Delta = \frac{1}{2} b h$$

base   height

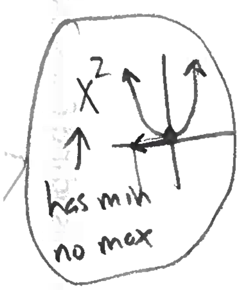
$$h = 10 + 3x$$

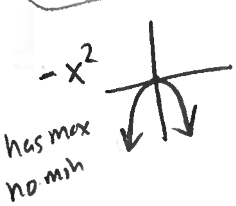
So,

$$\text{Area}_\Delta = \frac{1}{2} b h = \frac{1}{2} x (10 + 3x)$$

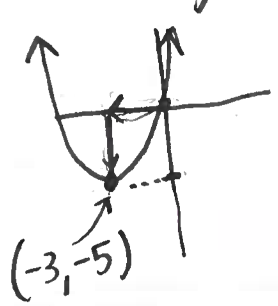
Ex: Given $y = (x+3)^2 - 5$ find min/max of this function.

y-values





Soln: By transformations, the graph of this is graph of x^2 shifted left by 3 + down by 5.



$\rightarrow$ $\boxed{\text{NO MAX}}$

min is -5 occurring at $x = -3$

Ex: A rocket is launched straight up with an initial velocity of 39 $\frac{ft}{s}$. The height h t seconds after launch is

$$\boxed{h(t) = -16t^2 + 39t + 14}$$

Find max height of the rocket.

Complete sq

$$x^2 + bx = \left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

$$4x^2 + 6 = 2(2x^2+3)$$

$\frac{4}{2}$ $\frac{6}{2}$

Soln: Complete the square:

$$-16\left(\underbrace{t^2 - \frac{39}{16}t}_{\text{"}t^2+bt\text{"}} - \frac{14}{16}\right)$$

where $b = -\frac{39}{16}$

$\frac{b}{2} = \frac{-39}{32}$

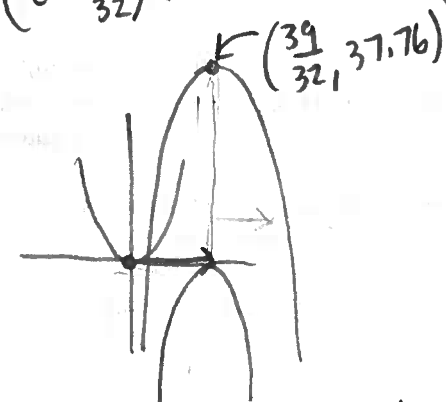
$$= -16\left[\left(t-\frac{39}{32}\right)^2 - \left(-\frac{39}{32}\right)^2 - \frac{14}{16}\right]$$

$-\left(\frac{-39}{32}\right)^2 - \frac{14}{16} \approx -2.36$

$$= -16\left(t-\frac{39}{32}\right)^2 - 16(-2.36)$$

$$= -16\left(t-\frac{39}{32}\right)^2 + 37.76$$

this is the function t^2 shifted right by $\frac{39}{32}$ + v. stretched + v. refl. by 16 + v. shifted up 37.76



The rocket reached a max height of 37.76 ft at time

$$t = \frac{39}{32} \approx 1.22 \text{ sec}.$$

Ex: The demand eqt for a product is

$$\boxed{p = 191 - 0.03x}$$

where p is price of product + x is number of units produced.

The total revenue obtained by producing + selling x units is

$$R = xp$$

Determine prices p that would yield a $\boxed{\text{revenue of } \$5000}$

Soln: $5000 = x(191 - 0.03x)$ ← has no "p" but that's ok

$$0 = 191x - 0.03x^2 - 5000$$

$$0 = -0.03x^2 + 191x - 5000$$

↓ solve (QF, $a = -0.03$, $b = 191$, $c = -5000$)

$ax^2 + bx + c = 0$

↓

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-191 \pm \sqrt{191^2 - 4(-0.03)(-5000)}}{2(-0.03)}$$

$$= \frac{-191}{2(-0.03)} \pm \frac{\sqrt{\text{mess}}}{-2(0.03)}$$

(+) $x = 26.28$ | (−) $x = 6340.38$

(can't make partial units)

$x = 27$ | $x = 6341$

$p = 191 - 0.03(27) = \$190.19$ → produce 27 units + charge \$190.19

$p = 191 - 0.03(6341) = \$0.77$ → produce 6341 units + charge 77 cents