

1

What is....

"What input to f gives output 6?"

$\rightarrow f^{-1}(6) = 5$

$f^{-1}(4) = 4$

"What input to f gives output of 2?"

$\rightarrow f^{-1}(2) = 3$

$f^{-1}(1) = 1$

Complex Numbers

2

"imaginary number"

$a(b+c)$

definition
 $i = \sqrt{-1}$

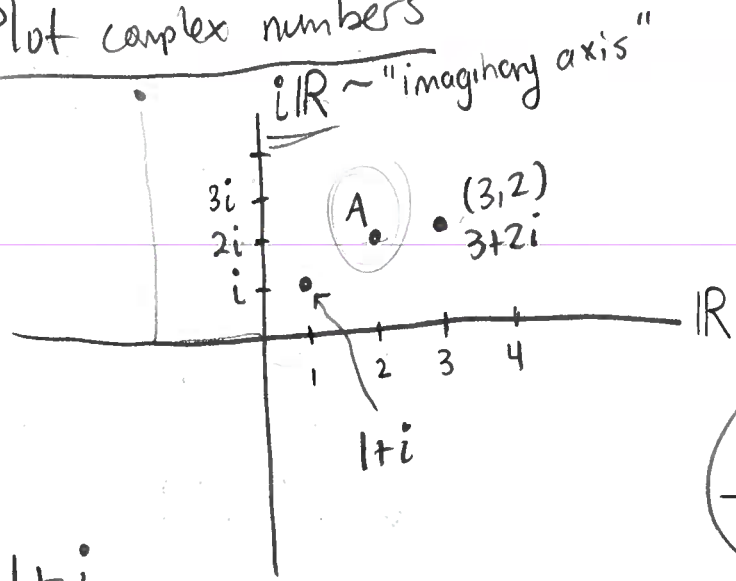
does not lie on number line!



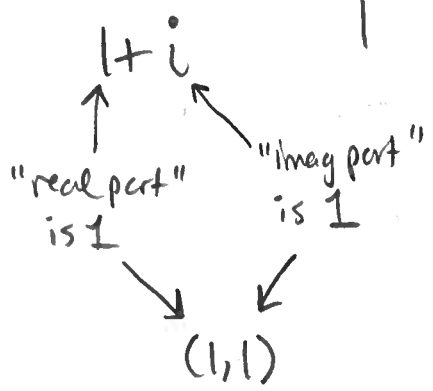
$i^2 = (\sqrt{-1})^2$
 $i^2 = -1$

Plot complex numbers

Complex plane
 \mathbb{C}



$-3+7i$



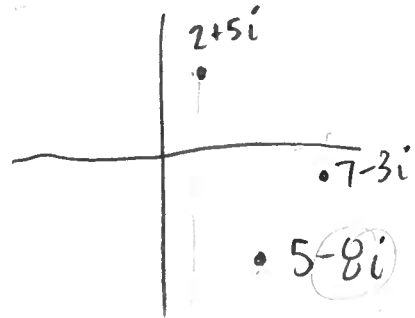
fact: a function $f: \mathbb{C} \rightarrow \mathbb{C}$
 cannot be graphed in a traditional way \rightarrow we won't try

Add complex #'s

(3)

Ex: $(5-8i) + (2+5i)$

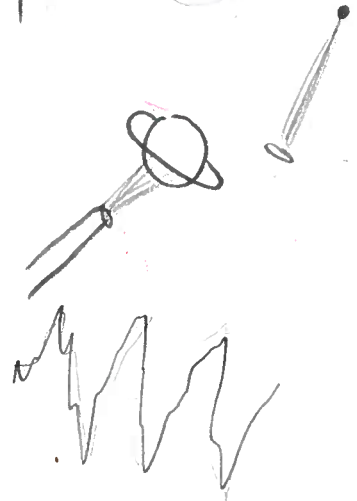
$$= (5+2) + (-8+5)i$$
$$= 7-3i$$



Subtract

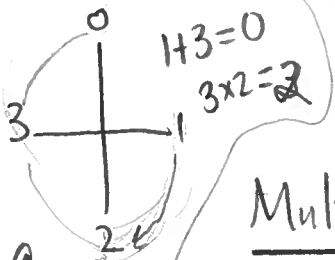
Ex: $(2-i) - (3+4i)$

$$= 2-i-3-4i$$
$$= (2-3) + (-1-4)i$$
$$= -1-5i$$



$2 \times 2 = 0$

$\{0, 1, 2, 3\}$



Multiplying

Ex: $(3+2i)(1-i) = (3+2i)(1) - (3+2i)(i)$

$$= 3+2i - 3i - 2i^2$$

$i^2 = -1$

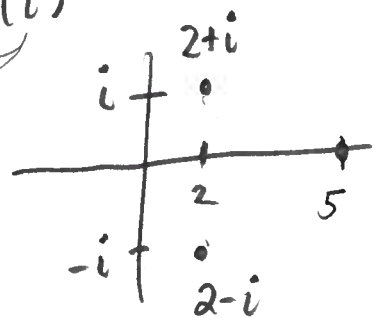
$$= (3+2) + (2-3)i$$
$$= 5-i$$

Ex: $(2-i)(2+i) = (2-i)(2) + (2-i)(i)$

$$= 4-2i+2i-i^2$$

$i^2 = -1$

$$= 4+1 = 5$$



Modular arithmetic

1

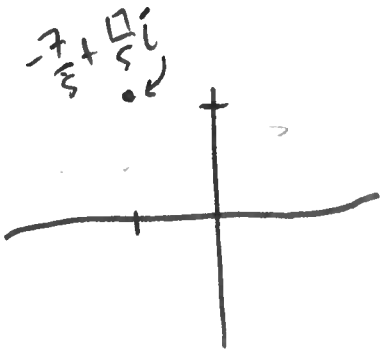
Division

Goal: write $\frac{a+bi}{c+di}$ in "standard form"
 $\#_1 + \#_2 i$

Ex: $\frac{5-i}{2+3i} = \frac{5-i}{2+3i} (1)$

"complex conjugate"
 $\overline{a+bi} = a-bi$
"complex conj ~ changes sign of $a+bi$ "
sign of the i term
 $\overline{2-3i} = 2+3i$

$(a-b)(a+b) = a^2 - b^2$



$$\begin{aligned} &= \frac{5-i}{2+3i} \left(\frac{\overline{2+3i}}{\overline{2+3i}} \right) \\ &= \frac{5-i}{2+3i} \left(\frac{2-3i}{2-3i} \right) \\ &= \frac{(5-i)(2) - (5-i)(3i)}{(2+3i)(2) - (2+3i)(3i)} \\ &= \frac{10-2i - (15i-3i^2)}{4+6i - (6i-9i^2)} \\ &= \frac{10-2i-15i+3}{4+6i-6i+9} \\ &= \frac{7-17i}{13} = \frac{-7}{5} + \frac{17}{5}i \end{aligned}$$

Ex: Write in std form $\overline{i-7} = -i-7$

5

$$\frac{1+i}{i-7} = \left(\frac{1+i}{i-7}\right)(1) = \left(\frac{1+i}{i-7}\right)\left(\frac{-i-7}{-i-7}\right)$$

$$= \frac{(1+i)(-i) - (1+i)(7)}{(i-7)(-i) - (i-7)(7)}$$

$$= \frac{-i - i^2 - 7 - 7i}{(i-7)(-i) - (i-7)(7)}$$

$$= \frac{-i - \overset{2}{i} - 7 - 7i}{(i-7)(-i) - (i-7)(7)}$$

$$= \frac{-i - i^2 - 7 - 7i}{-i^2 + 7i - 7i + 49}$$

$$= \frac{(-1-7)i + (1-7)}{(1+49)}$$

$$= \frac{-8i}{50} - \frac{6}{50}$$

$$= -\frac{6}{50} - \frac{8}{50}i$$

$$\begin{aligned} &-(i-7)(7) \\ &= -(7i-49) \\ &= -7i+49 \end{aligned}$$