

$$f(x) = \frac{1}{x-1}, \quad g(x) = \sqrt{2x+3}$$

$$\text{dom } f = \mathbb{R} \setminus \{1\}$$

$$\text{dom}(g) = \left[-\frac{3}{2}, \infty\right)$$

$$-\frac{3}{2} \leq x < \infty$$

$$-\frac{3}{2} \leq f(x) < \infty$$

$$0 < a < b \rightarrow 0 < \frac{1}{b} < \frac{1}{a}$$

$$0 < \frac{1}{b} < \frac{1}{a}$$

$$a < b < 0$$

$$-\frac{1}{b} < -\frac{1}{a} < 0$$

Need to make

Sure that

$$\begin{aligned} -2 < 0 < 2 \\ -\frac{1}{2} < 0 < \frac{1}{2} \end{aligned}$$

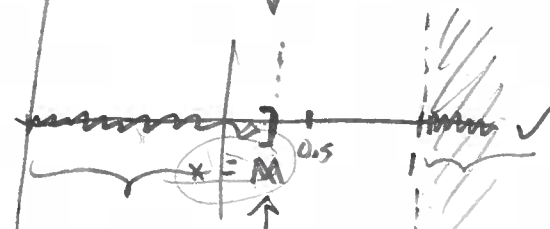
$$(g \circ f)(x) \stackrel{\text{def}}{=} g(f(x)) = g\left(\frac{1}{x-1}\right)$$

$$= \sqrt{2\left(\frac{1}{x-1}\right) + 3}$$

$$\Rightarrow \text{dom}(g \circ f) = \left(-\infty, \frac{1}{3}\right] \cup (1, \infty)$$

$$-\frac{3}{2} \leq \frac{1}{x-1} < \infty$$

desmos



Look at "="

$$-\frac{3}{2} = \frac{1}{M-1}$$

$$-\frac{2}{3} = M-1$$

$$M = \frac{1}{3}$$

By hand

$$\text{Suppose } x-1 < 0 \rightarrow x < 1$$

$$\Rightarrow \frac{1}{x-1} < 0$$

$$-\frac{3}{2} \leq \frac{1}{x-1} < 0 < \infty$$

$$x-1 \leq -\frac{2}{3} < 0 < \infty$$

$$x \leq -\frac{2}{3} + 1 = \frac{1}{3}$$

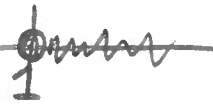
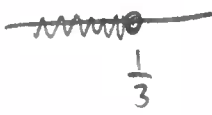
$$\text{Suppose } x \geq 1$$

$$x \neq 1 \text{ (ble of } f)$$

$$\begin{aligned} \Downarrow \\ x > 1 \\ \frac{1}{x-1} > 0 \end{aligned}$$

$$-\frac{3}{2} < 0 < \frac{1}{x-1} < \infty$$

$$-\frac{2}{3} < 0 < x-1 < \infty \rightarrow 1 < x$$



Ex: find fog, gof, dom f, dom g, dom fog, dom gof  
for

$$f(x) = \frac{1}{3x-1} \text{ and } g(x) = \sqrt{5x+2}$$

dom(f)

need  $3x-1 \neq 0$

$$3x \neq 1$$

~~no~~  
 $\frac{1}{3}$

$$x \neq \frac{1}{3}$$

$$\Rightarrow \text{dom}(f) = (-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty) \\ = \mathbb{R} \setminus \{\frac{1}{3}\}$$

dom(g)

make sure  $5x+2 \geq 0$

$$5x \geq -2$$

$$x \geq -\frac{2}{5}$$

$$\Rightarrow \text{dom}(g) = [-\frac{2}{5}, \infty)$$

$$-\frac{2}{5} \leq x < \infty$$

$$(g \circ f)(x) = g(f(x)) \\ = g\left(\frac{1}{3x-1}\right)$$

$$= \sqrt{\frac{5}{3x-1} + 2}$$

$$\Rightarrow -\frac{2}{5} \leq \frac{1}{3x-1} \leq \infty$$

$$\begin{aligned} 3x-1 < 0 &\Rightarrow x < \frac{1}{3} \\ -\frac{2}{5} \leq \frac{1}{3x-1} < \infty &\Rightarrow \frac{1}{3x-1} \geq -\frac{2}{5} \\ -\frac{5}{2} \geq 3x-1 &\Rightarrow 3x \leq -\frac{3}{2} \\ -\frac{3}{2} \geq 3x &\Rightarrow -\frac{1}{2} \geq x \end{aligned}$$

$$\begin{aligned} \frac{5}{3x-1} + 2 &\geq 0 \\ \frac{1}{3x-1} &\geq -\frac{2}{5} \\ 3x-1 &\geq 0 \\ x &\neq \frac{1}{3} \\ \Rightarrow x &> \frac{1}{3} \end{aligned}$$

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{5x+2})$$

$$= \frac{1}{3\sqrt{5x+2} - 1}$$

Need  $3\sqrt{5x+2} - 1 \neq 0$

$$3\sqrt{5x+2} - 1 \neq 0$$

$$\sqrt{5x+2} = \frac{1}{3}$$

$$5x+2 = \frac{1}{9} \\ 5x = \frac{1}{9} - 2 = -\frac{17}{9} \Rightarrow x = -\frac{17}{45}$$

$$\text{dom}(f \circ g) = [-\frac{2}{5}, -\frac{17}{45}) \cup (-\frac{17}{45}, \infty)$$