Homework 4 MATH 3520 Fall 2019

§3.1

#1) $A+2D = \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix} + 2\begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -6 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -5 & 7 \end{bmatrix}$

#4) $C - B^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 4 & -2 & 1 \\ 0 & 2 & 3 \end{bmatrix}^T$

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ -2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 5 & 2 \\ 4 & 3 \end{bmatrix}$

#9)

#11) $FE = \underbrace{\begin{bmatrix} -1 \\ 2 \end{bmatrix}}_{2\times 2} \underset{2\times 1}{\begin{bmatrix} 4 & 2 \end{bmatrix}} = \underbrace{(-1)(4) + 2(2)}_{\substack{1\times 1 \\ \text{i.e. a number}}} = 0$

#9) $E(AF) = [4 \ 2]\Big(\overbrace{\underset{2\times 2}{\begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} -1 \\ 2 \end{bmatrix}}}^{2\times 1}\Big)$

$= [4 \ 2]\begin{bmatrix} -3 \\ 11 \end{bmatrix} = -12 + 22 = 10$

#39) a) $a_{ij} = (-1)^{i+j} \Rightarrow a_{11} = (-1)^2 = 1, \ldots$

$$\begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

for example, this is the a_{23} entry

row col

$a_{23} = (-1)^{2+3} = (-1)^5 = -1$

§3.2

#4) $2(A-B+X)=3(X-A)$

$$2A-2B+2X=3X-3A$$

$$\boxed{5A-2B=X}$$

#14) We look at the equation

$$\alpha_1\begin{bmatrix}1&2\\4&3\end{bmatrix}+\alpha_2\begin{bmatrix}2&1\\-1&0\end{bmatrix}+\alpha_3\begin{bmatrix}1&1\\1&1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix}\alpha_1+2\alpha_2+\alpha_3 & 2\alpha_1+2\alpha_2+\alpha_3\\ 4\alpha_1-\alpha_2+\alpha_3 & 3\alpha_1+\alpha_3\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$\Downarrow$$

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$$\begin{cases}\alpha_1+2\alpha_2+\alpha_3=0\\ 2\alpha_1+2\alpha_2+\alpha_3=0\\ 4\alpha_1-\alpha_2+\alpha_3=0\\ 3\alpha_1+\alpha_3=0\end{cases}\Longrightarrow\left[\begin{array}{ccc|c}1&2&1&0\\2&2&1&0\\4&-1&1&0\\3&0&1&0\end{array}\right]\overset{\text{rref}}{\sim}\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{array}\right]$$

$\Longrightarrow$ soln vector

$$\vec{\alpha}=\begin{bmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$\Longrightarrow$ linearly independent

#37) a) $\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$

$\neq$ NOT EQUAL $\Rightarrow \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ not skew-symmetric

$-\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 2 & -3 \end{bmatrix}$

b) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$\parallel \xrightarrow{\text{equal}} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is skew-symmetric

$-\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

§3.3

#2) $\det \begin{bmatrix} 4 & -2 \\ 2 & 0 \end{bmatrix} = 0-(-4) = 4$

$\Rightarrow$ invertible

Using Thm 3.8: $\begin{bmatrix} 4 & -2 \\ 2 & 0 \end{bmatrix}^{-1} = \frac{1}{4}\begin{bmatrix} 0 & 2 \\ -2 & 4 \end{bmatrix}$

#3) $\det \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix} = 24-24 = 0$

$\Rightarrow$ NOT invertible (again see Thm 3.8)

#52

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & -2 & -1 & 0 & 1 & 0 \\ 2 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \overset{\text{rref}}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \underbrace{\begin{matrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 4 & 6 & -7 \end{matrix}}_{\text{this is the inverse}} \\ 0 & 0 & 1 & \end{array}\right]$$

You won't be so lucky to use software on exam ☺ !

#53

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 2 & 0 & 1 & 0 \\ 2 & 3 & -1 & 0 & 0 & 1 \end{array}\right] \overset{\text{rref}}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 1 & \\ 0 & 1 & -1 & \\ 0 & 0 & 0 & \end{array}\right]$$

NOT invertible!! Left piece not identity