1. Find the exact value:

   (a) $\displaystyle\sum_{k=0}^{\infty} \left(\frac{3}{4}\right)^k$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(k+1)(k+2)}$

2. For which values of x does the following sum converge?

$$\sum_{k=0}^{\infty} (5x)^k$$

3. Converge or diverge? Why? Explain, with appropriate calculations. (For full credit, be sure to state which theorem is used.)

   (a) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{\sqrt{k^2+5}}$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{k^3+17}$

   (c) $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{k^k}$

   (d) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2+5k+2}{k^2+2k+17}$

   (e) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$

   (f) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^\pi}$

   (g) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{5}{4}\right)^k$

   (h) $\displaystyle\sum_{k=1}^{\infty} \frac{4^k}{k!}$

   (i) $\displaystyle\sum_{k=1}^{\infty} \frac{k^k}{(2k+1)^k}$

4. (13 points) Find the exact value of the series.

   (a) (6 points) $\displaystyle\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k$

   (b) (7 points) $\displaystyle\sum_{k=2}^{\infty} \left(\frac{1}{k} - \frac{1}{k-1}\right)$

5. (21 points) Find the interval of convergence and radius of convergence of the following power series.

   (a) (7 points) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k x^k}{k}$

(b) (7 points) $\displaystyle\sum_{k=1}^{\infty} k! x^k$

(c) (7 points) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k x^k}{(k+1)!}$

6. (56 points) Converge or diverge? Why? Explain.

(a) (7 points) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k k}{k^2 - 5}$

(b) (7 points) $\displaystyle\sum_{k=1}^{\infty} \frac{k^4 - 3k^2 + 1}{2k^4 - k + 17}$

(c) (7 points) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^e}$

(d) (7 points) $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{(2k)!}$

(e) (7 points) $\displaystyle\sum_{k=1}^{\infty} \frac{k^k}{(3k+1)^k}$

(f) (7 points) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{10}{3}\right)^k$

(g) (7 points) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$

(h) (7 points) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^2 - 1}}$