§9.1

#5 $\sin(x)\cos(x)\sec(x) = \sin(x)\cancel{\cos(x)}\left(\frac{1}{\cancel{\cos(x)}}\right)$

$= \sin(x)$

#7 don't cancel!

$\tan(x)\sin(x) + \sec(x)\cos^2(x) = \frac{\sin(x)}{\cos(x)}\sin(x) + \left(\frac{1}{\cancel{\cos x}}\right)\cos^{\cancel{2}}(x)$

$= \frac{\sin^2(x)}{\cos(x)} + \frac{\cos^2(x)}{\cos(x)}$

$= \frac{\sin^2(x)+\cos^2 x}{\cos x} \leftarrow \boxed{\text{top} = 1}$

$= \frac{1}{\cos x}$

#8 $\csc(x) + \cos(x)\cot(-x) = \frac{1}{\sin x} + \cos(x)\frac{\cos(-x)}{\sin(-x)}$

$\boxed{\begin{array}{l}\cos(-x) = \cos(x)\\ \sin(-x) = -\sin(x)\end{array}} \rightarrow \; = \frac{1}{\sin(x)} - \frac{\cos^2(x)}{\sin(x)}$

$= \frac{1-\cos^2(x)}{\sin(x)} \leftarrow \boxed{\begin{array}{l}\text{from } \cos^2(x)+\sin^2(x) = 1\\ \text{you see}\\ \quad 1-\cos^2(x) = \sin^2(x)\end{array}}$

$= \frac{\sin^2(x)}{\sin(x)}$

$= \sin(x)$

#13

$$\frac{1+\tan^2(\theta)}{\csc^2(\theta)}+\sin^2(\theta)+\frac{1}{\sec^2(\theta)}=\frac{1+\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\sin^2\theta}}+\sin^2(\theta)+\frac{1}{\frac{1}{\cos^2(\theta)}}$$

$$=\underbrace{\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)}_{\left[=\frac{\cos^2\theta+\sin^2(\theta)}{\cos^2\theta}=\frac{1}{\cos^2\theta}\right]}\sin^2(\theta)+\underbrace{\sin^2(\theta)+\cos^2(\theta)}_{=1}$$

$$=\frac{\sin^2(\theta)}{\cos^2(\theta)}+1$$

$$=\frac{\sin^2(\theta)+\cos^2(\theta)}{\cos^2(\theta)}$$

$$=\frac{1}{\cos^2(\theta)}$$

$$=\sec^2(\theta)$$

#16

$$\frac{\tan(x)+\cot(x)}{\csc(x)}=\frac{\frac{\sin(x)}{\cos(x)}+\frac{\cos(x)}{\sin(x)}}{\frac{1}{\sin(x)}}=\frac{\frac{\sin^2(x)+\cos^2(x)}{\cos(x)\sin(x)}}{\frac{1}{\sin(x)}}$$

$$=\left(\frac{1}{\cos(x)\cancel{\sin(x)}}\right)\left(\frac{\cancel{\sin(x)}}{1}\right)$$

$$=\frac{1}{\cos(x)}$$

#17

$$\frac{\sec(x)+\csc(x)}{1+\tan(x)}=\frac{\frac{1}{\cos(x)}+\frac{1}{\sin(x)}}{1+\frac{\sin(x)}{\cos(x)}}=\frac{\frac{\sin(x)+\cos(x)}{\cos(x)\sin(x)}}{\frac{\cos(x)+\sin(x)}{\cos(x)}}$$

$$=\left(\frac{\cancel{\sin(x)+\cos(x)}}{\cancel{\cos(x)}\sin(x)}\right)\left(\frac{\cancel{\cos(x)}}{\cancel{\cos(x)+\sin(x)}}\right)$$

$$=\frac{1}{\sin(x)}$$

#32 Verify the identity

$$(\sin(x)+\cos(x))^2 = 1+2\sin(x)\cos(x)$$

Soln: Start with left:

$$(\sin(x)+\cos(x))^2 = \underbrace{\sin^2(x)}_{} + 2\sin(x)\cos(x) + \underbrace{\cos^2(x)}_{}$$

$=1$

use

$(a+b)^2 = a^2+2ab+b^2$

$$= 1+2\sin(x)\cos(x),$$

as was to be shown.

Back to the class

Section 9.1 #29: Prove the identity

$$\cos(x)-\cos^3(x)=\cos(x)\sin^2(x).$$

Solution: Recall the Pythagorean identity $\cos^2(x)+\sin^2(x)=1$ and rearrange it to say $\cos^2(x)=1-\sin^2(x)$. Start with the left and calculate

$$\begin{aligned}\cos(x)-\cos^3(x) &= \cos(x)(1-\cos^2(x)) \\ &= \cos(x)\sin^2(x),\end{aligned}$$

completing the proof. ■

Section 9.1 #30: Prove the identity

$$\cos(x)(\tan(x)-\sec(-x))=\sin(x)-1.$$

Solution: Recall the "even property" of cosine, i.e. $\cos(-x)=\cos(x)$. Start with the left and calculate

$$\begin{aligned}\cos(x)(\tan(x)-\sec(-x))=\sin(x)-1 &= \cos(x)\left(\frac{\sin(x)}{\cos(x)}-\frac{1}{\cos(-x)}\right)=\sin(x)-1 \\ &= \cos(x)\left(\frac{\sin(x)}{\cos(x)}-\frac{1}{\cos(x)}\right) \\ &= \cos(x)\left(\frac{\sin(x)-1}{\cos(x)}\right) \\ &= \sin(x)-1,\end{aligned}$$

completing the proof. ■

Section 9.1 #33: Prove the identtiy

$$\cos^2(x)-\tan^2(x)=2-\sin^2(x)-\sec^2(x).$$

Solution: Recall from the Pythagorean identity $\cos^2(x)+\sin^2(x)=1$ we may conclude both $1-\sin^2(x)=\cos^2(x)$ and $\cos^2(x)-1=-\sin^2(x)$. Start with the right and calculate

$$\begin{aligned}
2-\sin^2(x)-\sec^2(x) &= 2-\sin^2(x)-\frac{1}{\cos^2(x)} \\
&= (1-\sin^2(x))+1-\frac{1}{\cos^2(x)} \\
&= \cos^2(x)+\frac{\cos^2(x)-1}{\cos^2(x)} \\
&= \cos^2(x)+\frac{-\sin^2(x)}{\cos^2(x)} \\
&= \cos^2(x)-\tan^2(x),
\end{aligned}$$

completing the proof. ■