MATH 1540 - EXAM 2 FALL 2019

SOLUTION

Friday, 18 October
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (12 points) Compute...

   (a) (4 points) $\cot\left(\frac{\pi}{2}\right)$

   Solution: Using the unit circle, compute

$$\cot\left(\frac{\pi}{2}\right) = \frac{\cos\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)} = \frac{0}{1} = 0$$

   (b) (4 points) $\sec\left(\frac{3\pi}{4}\right)$

   Solution: Using the unit circle, compute

$$\sec\left(\frac{3\pi}{4}\right) = \frac{1}{\cos\left(\frac{3\pi}{4}\right)} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}}$$

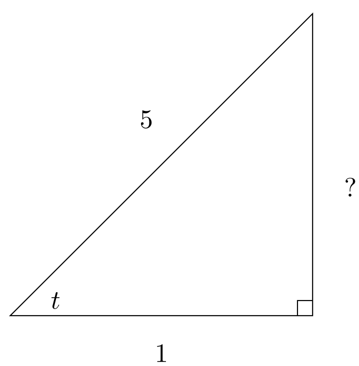
   (c) (4 points) $\csc\left(\frac{5\pi}{6}\right)$

   Solution: Using the unit circle, compute

$$\csc\left(\frac{5\pi}{6}\right) = \frac{1}{\sin\left(\frac{5\pi}{6}\right)} = \frac{1}{\frac{1}{2}} = 2.$$

2. (15 points) Given that $\cos(t) = -\frac{1}{5}$ and t is in Quadrant III, find $\sin(t)$, $\tan(t)$, $\csc(t)$, $\sec(t)$, and $\cot(t)$.

   Solution: First draw a triangle:



   We can find the missing leg using Pythagorean theorem:

$$1^2 + ?^2 = 5^2,$$

   hence

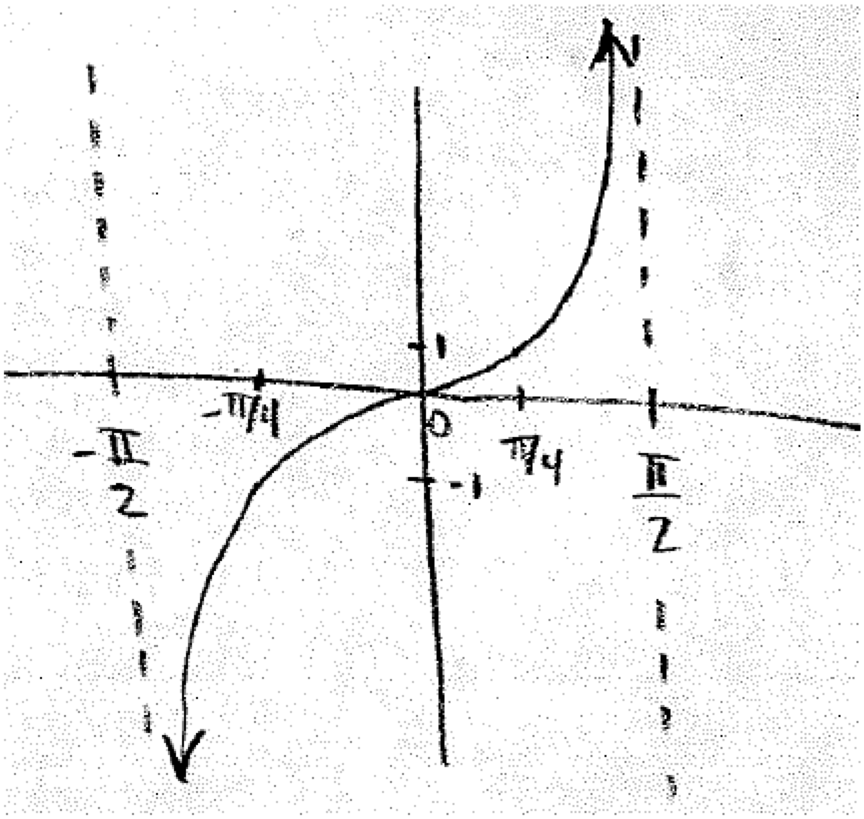
$$? = \sqrt{24}.$$

   Since t is in Quadrant III, $\sin(t) = -\frac{\sqrt{24}}{5}$. We can now easily compute the remaining trig functions:

$$\tan(t) = \frac{\sin(t)}{\cos(t)} = \frac{-\frac{\sqrt{24}}{5}}{-\frac{1}{5}} = \sqrt{24}, \quad \cot(t) = \frac{1}{\tan(t)} = \frac{1}{\sqrt{24}},$$

$$\csc(t) = \frac{1}{\sin(t)} = -\frac{5}{\sqrt{24}}, \qquad \sec(t) = \frac{1}{\cos(t)} = -5.$$

3. (15 points) Draw the graph of $y = \tan(x)$.
Solution: The anchor points we use are the standard ones for the tangent function: $-\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}$.



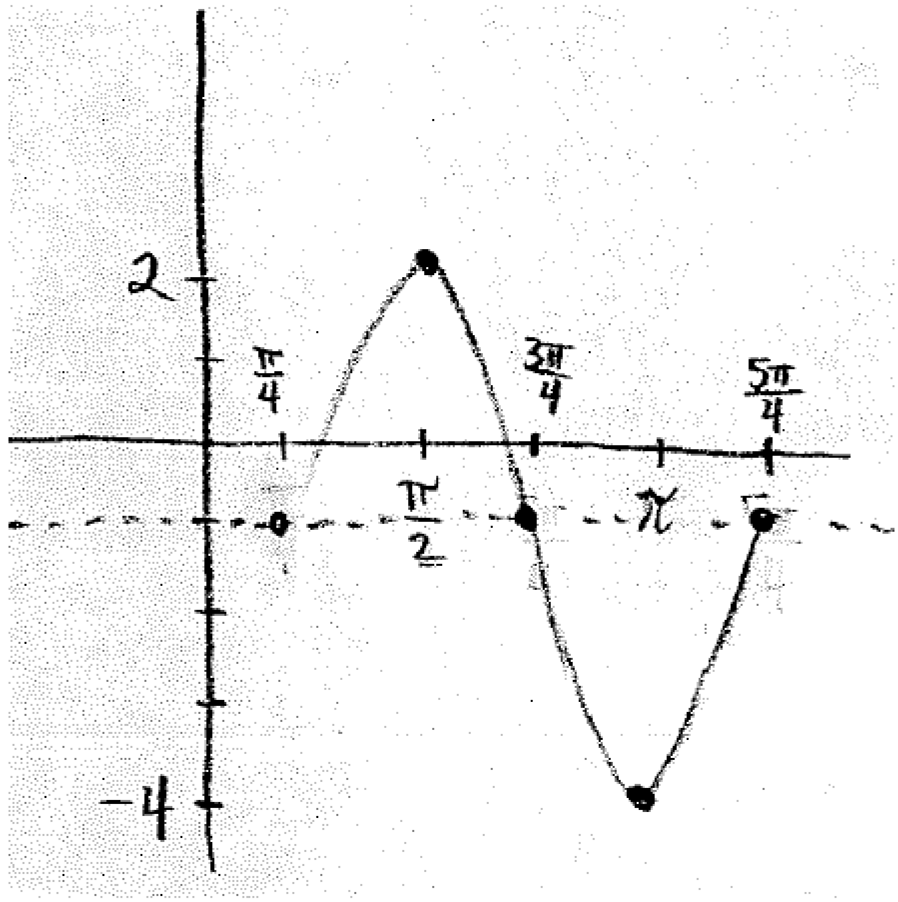
4. (15 points) Draw the graph of $y = 3\sin\left(2\left(x - \frac{\pi}{4}\right)\right) - 1$.
Solution: The "3" multiplies y-values, the "2" divides x-values, the "$\frac{\pi}{4}$" is a horizontal shift right, and the "-1" is a vertical shift down.

Anchor points

$$\begin{array}{ccccc}
0 & \frac{\pi}{2} & \pi & \frac{3\pi}{2} & 2\pi \\
\downarrow & & \downarrow & & \downarrow \text{ div by 2} \\
0 & \frac{\pi}{4} & \frac{\pi}{2} & \frac{3\pi}{4} & \pi \\
\downarrow & & \downarrow & & \downarrow \text{ add } \frac{\pi}{4} \\
\frac{\pi}{4} & \frac{\pi}{2} & \frac{3\pi}{4} & \pi & \frac{5\pi}{4}
\end{array}$$

***y*-values**

$$\begin{array}{ccccc}
1 & & 3 & & 2 \\
\Big| & \underline{\text{mult by 3}} & \Big| & \underline{\text{subtr 1}} & \Big| \\
-1 & & -3 & & -4
\end{array}$$



5. (19 points) Find the exact value of...

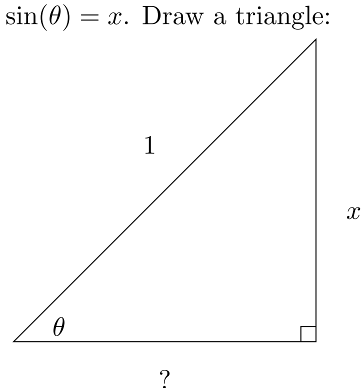
(a) (5 points) $\tan^{-1}(1)$

Solution: From the unit circle, since the point at $\frac{\pi}{4}$ is in quadrant I and has coordinates $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, we conclude that

$$\tan^{-1}(1) = \frac{\pi}{4}.$$

(b) (7 points) $\cos\left(\sin^{-1}(x)\right)$

Solution: Let $\theta = \sin^{-1}(x)$, then $\sin(\theta) = x$. Draw a triangle:



Find the missing leg of the triangle using Pythagorean theorem: $?^2 + x^2 = 1^2$, hence $? = \sqrt{1-x^2}$.

Therefore, we may compute

$$\cos(\sin^{-1}(x)) = \cos(\theta) = \sqrt{1 - x^2}.$$

(c) (7 points) $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right)$

Solution: In this case, we cannot simply cancel $\sin^{-1}$ with sin because the range of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Since $\sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$, we need to find which angle in the allowed range has sine $-\frac{\sqrt{2}}{2}$. The answer is

$$\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right) = -\frac{\pi}{4},$$

which occupies the same *position* as $\frac{7\pi}{4}$, but has the correct *value* to lie in the appropriate range.

Class points_____/______ **Arizona Rubric Score ___________** **ver A** **Place Name on Back**

Scale for Rubric Score 4: 22-24 3: 17-21 2: 12-16 1:7 -11 0: 0-6

Outcome 3: Use the language of mathematics to determine relationships and patterns in graphs and characteristics of circular trig functions using pre-requisite knowledge of graphing techniques of common functions.

Math 1540 Assessment Outcome 3 Fall 2019

1. *(12 pts.)* **In a full sentence, describe precisely the effect of each value(number) in the equation when the circular function is graphed. All terms in the word bank must be used at least once.**

WORD BANK	*phase shift*	*period*	*amplitude*	*vertical shift*	*reflect in x-axis*	*reflect in y-axis*

$$f(x) = -7\cos(4(x - \frac{\pi}{2})) + 30 \quad \text{which is equivalent to} \quad f(x) = -7\cos(4x - 2\pi)) + 30$$

7
Negative sign on the 7
4
$\frac{\pi}{2}$
30
What if the "4" was "negative 4" instead of positive?

2. *(12pts. total)* **Write the letter of the equation on its matching graph. Scale for the y axis is one per gridline.**

A. $y = -2\sin(x)$

B. $y = \sin(x) - \frac{\pi}{2}$

C. $y = 2\sin(x)$

D. $y = \sin(x - \frac{\pi}{2})$

E. $y = \sin(2x)$

F. $y = \sin(x) + 2$