MATH 1540 - EXAM 1 FALL 2019
SOLUTION

Wednesday, 18 September
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (15 points) (a) (5 points) Convert 23° to radians.
   Solution: Calculate
$$23^\circ = (23^\circ)\left(\frac{2\pi \text{ rad}}{360^\circ}\right) = \frac{23\pi}{180} \text{ radians}$$

   (b) (5 points) Convert $\frac{\pi}{11}$ radians to degrees.
   Solution: Calculate
$$\frac{\pi}{11} \text{ radians} = \left(\frac{\pi}{11} \text{ radians}\right)\left(\frac{360^\circ}{2\pi \text{ radians}}\right) = \left(\frac{360\pi}{22\pi}\right)^\circ = \left(\frac{180}{11}\right)^\circ.$$

   (c) (5 points) Convert 5 radians to degrees.
   Solution: Calculate
$$5 \text{ radians} = (5 \text{ radians})\left(\frac{360^\circ}{2\pi \text{ radians}}\right) = \left(\frac{1800}{2\pi}\right)^\circ = \left(\frac{900}{\pi}\right)^\circ.$$

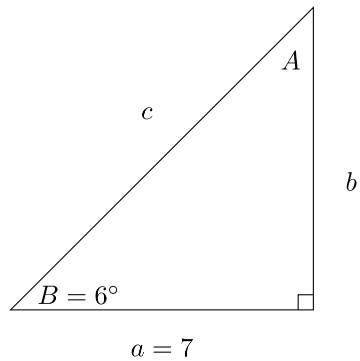
2. (20 points) Find an exact value for
   (a) (5 points) $\cos(0)$
   Solution: From the unit circle,
$$\cos(0) = 1.$$

   (b) (5 points) $\sin\left(\frac{\pi}{3}\right)$
   Solution: From the unit circle,
$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}.$$

   (c) (5 points) $\cos\left(\frac{5\pi}{4}\right)$
   Solution: From the unit circle,
$$\cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}.$$

   (d) (5 points) $\sin\left(\frac{3\pi}{2}\right)$
   Solution: From the unit circle,
$$\sin\left(\frac{3\pi}{2}\right) = -1.$$

3. (10 points) Find all unknown sides of the given triangle in terms of the given information.



Solution:

Find b

Here we need a trigonometric function that relates the known angle $6°$ iwth the known side $a = 7$ and the unknown side b. Since b is the opposite of B and a is adjacent to B, we will use the tangent function to compute

$$\tan(6°) = \frac{b}{7}.$$

To solve this for b, multiply by 7 to get

$$b = 7\tan(6°).$$

Find c

We could use the side we just found with a trigonometric function or use the Pythagorean theorem here, but it is easier to simply use a a trigonometric function that relates the angle B to its adjacent a and the hypotenuse c. In other words, we want to use the cosine function:

$$\cos(6°) = \frac{7}{c},$$

and to solve it, first multiply by c on both sides to get

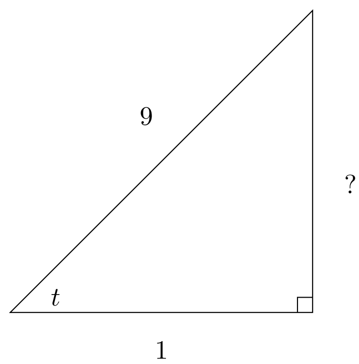
$$c\cos(6°) = 7.$$

Now divide by the number $\cos(6°)$ to get

$$c = \frac{7}{\cos(6°)}.$$

4. (10 points) If $\cos(t) = -\frac{1}{9}$ and t is in quadrant III, then find $\sin(t)$.

Solution: First we draw a triangle fitting $\cos(t) = \frac{1}{9}$ — ***NOTE: although the cosine here is negative, we cannot use negative numbers as side lengths of a triangle. So we instead will determine the sign of*** sin ***using the quadrant of*** t***.***



To find the side labelled "?", we use the Pythagorean theorem to see

$$1^2 + ?^2 = 9^2.$$

Solving this equation for $?^2$ and simplifying the arithmetic yields

$$?^2 = 80.$$

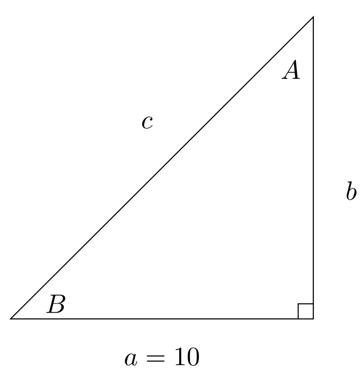
Taking the square root (and immediately discarding the negative root, since side lengths are never negative) yields

$$? = \sqrt{80}.$$

Since t is in quadrant III, we conclude that $\sin(t)$ is a negative number. Therefore, we compute

$$\sin(t) = -\frac{\sqrt{80}}{9}.$$

5. (10 points) If $a = 10$ and $\sin(B) = \frac{1}{3}$, then find all side lengths of the triangle:



Solution: Using the definiton of $\sin(B)$ we observe that

$$(*) \qquad \underbrace{\frac{1}{3}}_{\text{given}} = \sin(B) = \underbrace{\frac{b}{c}}_{\text{definition}} .$$

Since the equation $\frac{1}{3} = \frac{b}{c}$ has two variables, it is impossible to determine which values they take (***NOTE: many people immediately assigned*** $b = 1$ ***and*** $c = 3$***, and while this seems to work from the equation, such values do not define a triangle! To see that it doesn't work, try***

to use Pythagorean theorem on the triangle you would end up with – it will fail.) Using the Pythagorean theorem, we have

$$10^2 + b^2 = c^2,$$

and solving for c (and throwing away the negative root), we get

$$(**) \qquad c = \sqrt{100 + b^2}.$$

Plugging this equation into $(*)$ yields the equation

$$\frac{1}{3} = \frac{b}{\sqrt{100 + b^2}}.$$

To solve this, multiply by $\sqrt{100 + b^2}$ to get

$$\frac{\sqrt{100 + b^2}}{3} = b.$$

To make things easier, multiply by 3 to get

$$\sqrt{100 + b^2} = 3b.$$

Now square both sides and arraive at

$$100 + b^2 = 9b^2.$$

Subtract b^2 to get

$$100 = 8b^2.$$

Divide by 8 to get

$$\frac{100}{8} = b^2,$$

and finally take the square root to get

$$b = \frac{10}{\sqrt{8}}.$$

Now using $(**)$, we obtain c by plugging this value of b in:

$$c = \sqrt{100 + \left(\frac{10}{\sqrt{8}}\right)^2} = \sqrt{100 + \frac{100}{8}}.$$

6. (7 points) If an angle of $\theta = 11°$ subtends an arc length of 8 inches in a circle, then what is the radius of that circle?
Solution: We want to use the formula $s = r\theta$ (we are told $\theta = 11°$ and $s = 8$), but it requires the angle to be in radians. First make that conversion:

$$11° = (11°)\left(\frac{2\pi \text{ radians}}{360°}\right) = \frac{22\pi}{360} \text{ radians} = \frac{11\pi}{180} \text{ radians}.$$

Now using the equation $s = r\theta$ with the known values, we get

$$8 = r\left(\frac{11\pi}{8}\right).$$

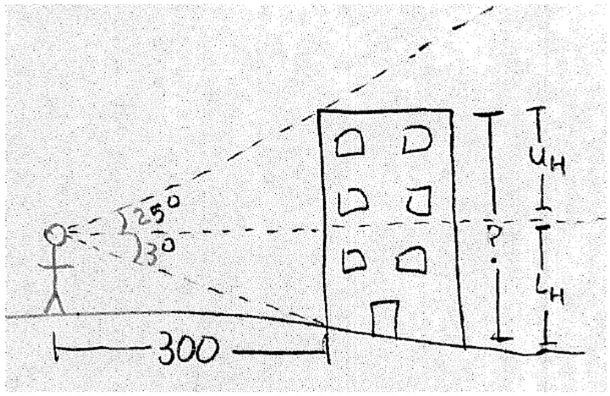
Multiply by 8 to get

$$64 = 11\pi r,$$

and divide by 11π to get

$$r = \frac{64}{11\pi}.$$

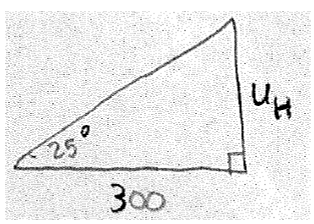
7. (10 points) A person standing 300 feet away from a building measures the angle of elevation to the top of the building to be 25° and measures the angle of depression to the bottom of the building to be 3°. What is the height of the building? Express your answer accurate to four decimal places.
Solution: Draw a picture:

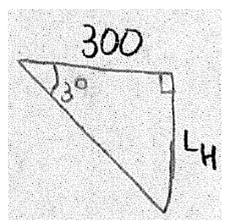


From the picture, it is evident that the desired height (labelled "?") can be expressed in terms of the heights labelled U_H ("upper height") and L_H ("lower height"):

$$? = U_H + L_H.$$

From the picture we extract two triangles:





In the first one, we use the tangent function to write

$$\tan(25°) = \frac{U_H}{300},$$

and then multiply by 300 to obtain

$$U_H = 300\tan(25°) \approx 139.9.$$

Similarly, we use the tangent function in the second picture to write

$$\tan(3°) = \frac{L_H}{300},$$

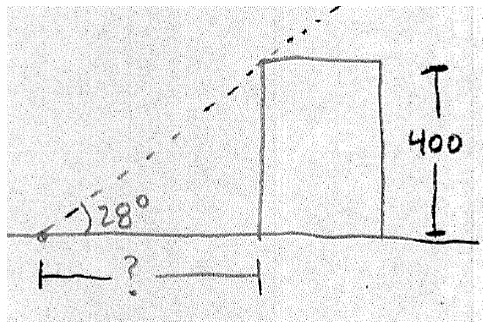
and multiplying by 300 yields

$$L_H = 300\tan(3°) \approx 15.72.$$

Therefore, the height of the building, labelled "?", is given by

$$? = U_H + L_H \approx 139.9 + 15.72 = 155.62 \text{ ft}.$$

8. (10 points) From the ground, the angle of elevation to the top of a 400 ft tall building is 28°. How far away from the building was this measurement taken? Express your answer accurate to four decimal places.
Solution: Draw a picture of this situation:



In the picture, the distance from the building is labelled "?". From this, we see that using the tangent function yields

$$\tan(28^\circ) = \frac{400}{?}.$$

To solve for ?, multiply by it to get

$$?\tan(28^\circ) = 400,$$

and then divide by then number $\tan(28^\circ)$ to get

$$? = \frac{400}{\tan(28^\circ)} \approx 752 \text{ ft.}$$

9. (8 points) Calculate and express your answer accurate to four decimal places.

   (a) (4 points) $\sin(4 \text{ rad})$
   Solution:

   $$\sin(4 \text{ rad}) \approx -0.7568.$$

   (b) (4 points) $\cos(36^\circ)$
   Solution:

   $$\cos(36^\circ) \approx 0.8090.$$