

$$\cos^2(x) + \sin^2(x) = 1$$

$$\cos^2(x) = 1 - \sin^2(x)$$

①

Section 9.1 #20

$$\frac{1}{1-\cos(x)} - \frac{\cos(x)}{1+\cos(x)} = \frac{(1+\cos(x)) - (1-\cos(x))\cos(x)}{(1-\cos(x))(1+\cos(x))}$$

$$= \frac{1 + \cancel{\cos(x)} - \cancel{\cos(x)} + \cos^2(x)}{1 - \cos^2(x)}$$

$$= \frac{1 + \cos^2(x)}{1 - \cos^2(x)} = \frac{1 + (1 - \sin^2(x))}{\sin^2(x)} = 2 \csc^2(x) - 1$$

#21) $(\sec(x) + \csc(x))(\sin(x) + \cos(x)) - 2 - \cot(x)$

$$= \left(\frac{1}{\cos(x)} + \frac{1}{\sin(x)} \right) (\sin(x) + \cos(x)) - 2 - \frac{\cos(x)}{\sin(x)}$$

$$= \frac{\sin(x) + \cos(x)}{\cos(x)\sin(x)} (\sin(x) + \cos(x)) - 2 - \frac{\cos(x)}{\sin(x)}$$

$$= \frac{\sin(x) + \cos(x)}{\cos(x)} + \frac{\sin(x) + \cos(x)}{\sin(x)} - 2 - \frac{\cos(x)}{\sin(x)}$$

$$= \left(\frac{\sin(x)}{\cos(x)} + 1 \right) + \left(1 + \frac{\cos(x)}{\sin(x)} \right) - 2 - \frac{\cos(x)}{\sin(x)}$$

$$= \frac{\sin(x)}{\cos(x)} = \tan(x)$$

#22) $\frac{1}{\csc(x) - \sin(x)} = \frac{1}{\frac{1}{\sin(x)} - \sin(x)} = \frac{1}{\frac{1 - \sin^2(x)}{\sin(x)}} = \frac{\sin(x)}{1 - \sin^2(x)}$

$$= \frac{\sin(x)}{\cos^2(x)}$$

$$= \left(\frac{\sin(x)}{\cos(x)} \right) \left(\frac{1}{\cos(x)} \right)$$

$$= \tan(x) \sec(x)$$

#29) Start w/ left:

$$\begin{aligned} \cos(x) - \cos^3(x) &= \cos(x) \overbrace{[1 - \cos^2(x)]}^{= \sin^2(x)} \\ &= \cos(x) \sin^2(x) \end{aligned}$$

#30) Start w/ left:

$$\begin{aligned} \cos(x) (\tan(x) - \sec(-x)) &= \cos(x) \left[\frac{\sin(x)}{\cos(x)} - \frac{1}{\cos(-x)} \right] \quad \leftarrow \cos(-x) = \cos(x) \\ &= \frac{\sin(x)}{\cancel{\cos(x)}} \cancel{\cos(x)} - \frac{1}{\cancel{\cos(x)}} \cancel{\cos(x)} \\ &= \sin(x) - 1 \end{aligned}$$

#32) Start w/ left:

$$\begin{aligned} (\sin(x) + \cos(x))^2 &= (\sin(x) + \cos(x))(\sin(x) + \cos(x)) \\ &= \sin^2(x) + \overbrace{2\sin(x)\cos(x)} + \cos^2(x) \\ &= 1 + 2\sin(x)\cos(x) \end{aligned}$$

#33) Start w/ right:

$$\begin{aligned} 2 - \sin^2(x) - \sec^2(x) &= 2 - (1 - \cos^2(x)) - (\tan^2(x) + 1) \\ &= \cancel{2} - 1 + \cos^2(x) - \tan^2(x) - \cancel{1} \\ &= \cos^2(x) - \tan^2(x) \end{aligned}$$

$$\begin{aligned} \sin^2(x) + \cos^2(x) &= 1 \quad \xrightarrow{\text{div by } \cos^2(x)} \\ \sin^2(x) &= 1 - \cos^2(x) \quad \downarrow \\ \tan^2(x) + 1 &= \sec^2(x) \quad \downarrow \\ &= \end{aligned}$$

Problem A Start w/ left

(3)

$$\frac{1 - \sin^2(x)}{\cos(x)} = \frac{\cos^2(x)}{\cos(x)} = \cos(x)$$

Problem B Start w/ right:

$$\sec(x) - \cos(x) = \frac{1}{\cos(x)} - \cos(x) = \frac{1 - \cos^2(x)}{\cos(x)} = \frac{\sin^2(x)}{\cos(x)}$$

~~$\cos(x)$~~

Problem C: Start w/ left:

$$\frac{\cos(x)}{\sin(x)\cot(x)} = \frac{\cos(x)}{\sin(x)\left(\frac{\cos(x)}{\sin(x)}\right)} = \frac{\cos(x)}{\sin(x)} \cdot \frac{\sin(x)}{\cos(x)} = 1$$