HW11 MATH 1540 FALL 2018

§9.2 #47 Start w/ left

$$\tan\left(x+\frac{\pi}{4}\right)=\frac{\sin\left(x+\frac{\pi}{4}\right)}{\cos\left(x+\pi/4\right)}=\frac{\sin(x)\cos(\pi/4)+\cos(x)\sin(\pi/4)}{\cos(x)\cos(\pi/4)-\sin(x)\sin(\pi/4)}$$

$$=\frac{\left(\sqrt{2}/2\right)\sin(x)+\left(\frac{\sqrt{2}}{2}\right)\cos(x)}{\left(\sqrt{2}/2\right)\cos(x)-\left(\sqrt{2}/2\right)\sin(x)}$$

$$=\left(\frac{\sqrt{2}/2}{\sqrt{2}/2}\right)^{1}\left[\frac{\sin(x)+\cos(x)}{\cos(x)-\sin(x)}\right]$$

multiply by $1=\dfrac{\frac{1}{\cos x}}{\frac{1}{\cos(x)}}$

$$=\left(\frac{\sin(x)+\cos(x)}{\cos(x)-\sin(x)}\right)\left(\frac{\frac{1}{\cos(x)}}{\frac{1}{\cos(x)}}\right)$$

$$=\frac{\frac{\sin(x)+\cos(x)}{\cos(x)}}{\frac{\cos(x)-\sin(x)}{\cos(x)}}=\frac{\tan(x)+1}{1-\tan(x)}$$

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$$\frac{\cos(x+h)-\cos(x)}{h}=\frac{\cos(x)\cos(h)-\sin(x)\sin(h)-\cos(x)}{h}$$

$$=\cos(x)\left[\frac{\cos(h)-1}{h}\right]-\sin(x)\left[\frac{\sin(h)}{h}\right]$$

§9.3 | #55 | Start w/ right

$$\frac{2\tan(x)}{1+\tan^2(x)} = \frac{2\frac{\sin(x)}{\cos(x)}}{1+\frac{\sin^2 x}{\cos^2 x}} = \frac{2\frac{\sin x}{\cos x}}{\frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)}} \quad (\cos^2(x)+\sin^2(x)=1)$$

$$= 2\left(\frac{\sin x}{\cos x}\right)\left(\frac{\cos^2 x}{1}\right)$$

$$= 2\sin(x)\cos(x)$$

$$= \sin(2x)$$

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$$\sin(3x) = \sin(x+2x) = \sin(x)\cos(2x) + \cos(x)\sin(2x)$$

$$= \sin(x)\left[\cos^2(x)-\sin^2(x)\right] + \cos(x)\left[2\sin(x)\cos(x)\right]$$

$$= \sin(x)\cos^2(x) - \sin^3(x) + 2\sin(x)\cos^2(x)$$

$$= 3\sin(x)\cos^2(x) - \sin^3(x)$$

Problem A:

(★) $\cos\left(\sin^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(-\frac{1}{7}\right)\right) = \cos\left(\sin^{-1}\left(\frac{1}{4}\right)\right)\cos\left(\tan^{-1}\left(-\frac{1}{7}\right)\right)$

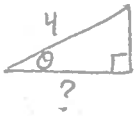
$$- \sin\left(\sin^{-1}\left(\frac{1}{4}\right)\right)\sin\left(\tan^{-1}\left(-\frac{1}{7}\right)\right)$$

($\sin^{-1}\left(\frac{1}{4}\right) = \frac{1}{4}$)

Compute $\cos(\sin^{-1}(\frac{1}{4}))$ and

Let $\theta = \sin^{-1}(\frac{1}{4}) \rightarrow \theta$ in QI or QIV

$\sin(\theta) = \frac{1}{4} \rightarrow \theta$ in QI or QII



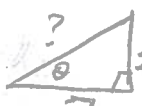
$? \rightarrow ?^2+1^2=4^2 \rightarrow ? = \sqrt{15}$

$\Rightarrow \cos(\sin^{-1}(\frac{1}{4})) = \cos(\theta) = \frac{\sqrt{15}}{4}$

Compute $\cos(\tan^{-1}(-\frac{1}{7}))$ and $\sin(\tan^{-1}(-\frac{1}{7}))$

Let $\theta = \tan^{-1}(-\frac{1}{7}) \rightarrow \theta$ in QI ~~OR~~ QIV

$\tan(\theta) = -\frac{1}{7} \rightarrow \theta$ in QII or QIV



$7^2+1^2=?^2 \rightarrow ? = \sqrt{50}$

$\Rightarrow \cos(\tan^{-1}(-\frac{1}{7})) = \cos(\theta) = +\frac{7}{\sqrt{50}}$

and $\sin(\tan^{-1}(-\frac{1}{7})) = \sin(\theta) = -\frac{1}{\sqrt{50}}$

Therefore by (★) we have

$$\cos\left(\sin^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(-\frac{1}{7}\right)\right) = \left(\frac{\sqrt{15}}{4}\right)\left(\frac{7}{\sqrt{50}}\right) - \left(\frac{1}{4}\right)\left(-\frac{1}{\sqrt{50}}\right)$$

$$= \frac{7\sqrt{15}+1}{4\sqrt{50}}$$