MATH 1540 - EXAM 2 - FALL 2018
SOLUTION

12 October 2018
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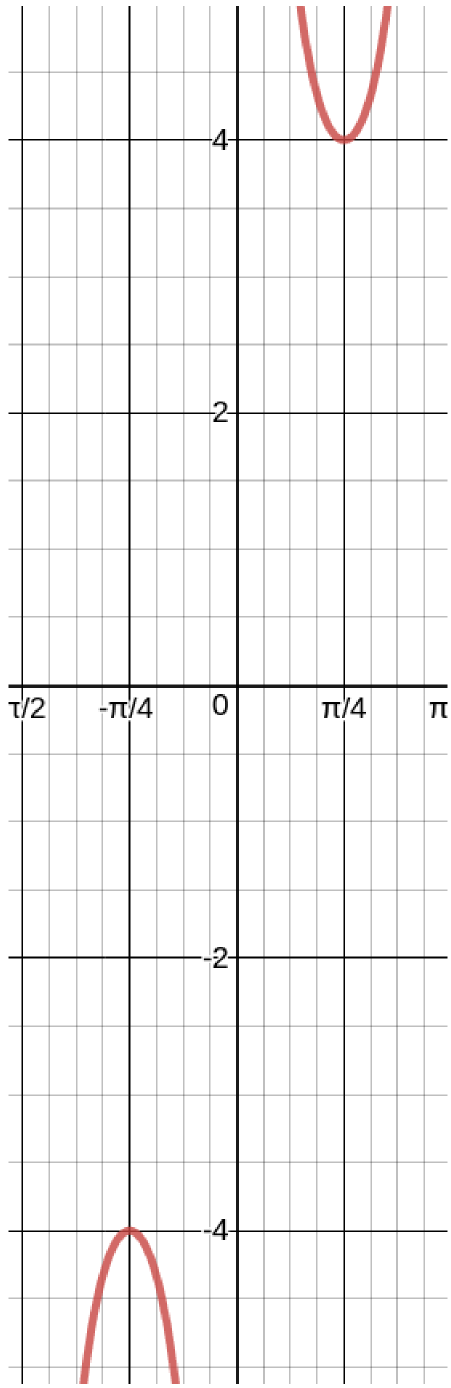
Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (16 points) Plot $y = 4\csc(2x)$.
Solution: First note that $y = \csc(x)$ has asymptotes at $-\pi, 0, \pi$, and so the starting anchor points to plot this are $-\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi$. The "$2x$" is a horizontal compression (action: divide anchor points by 2) and the "4" is a vertical stretch (action: multiply y-values by 4). Therefore the anchor points become $-\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}$. The y-values are multiplied by 4. Now we plot it:



2. (12 points) Find the exact value.

   (a) (4 points) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

   Solution: This problem is asking us to find the angle in quadrants I or II whose cosine is $\frac{\sqrt{2}}{2}$. This angle is $\frac{\pi}{4}$.

   (b) (4 points) $\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

   Solution: First compute

$$\tan\left(\frac{3\pi}{4}\right) = \frac{\sin\left(\frac{3\pi}{4}\right)}{\cos\left(\frac{3\pi}{4}\right)} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1.$$

   Now we see that we are being asked

$$\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right) = \tan^{-1}(-1).$$

   This is asking us to find the angle, in quadrant I or IV (where QIV is interpreted with negative angles), whose tangent is -1. The answer is $-\frac{\pi}{4}$.

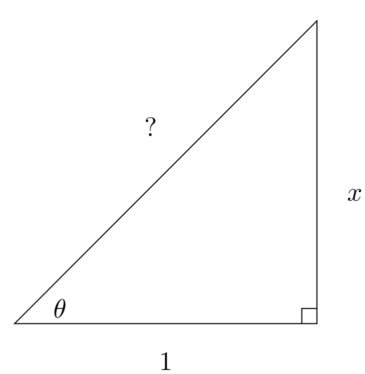
   (c) (4 points) $\sin^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right)$

   Solution: First calculate $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$. Now we see that we are being asked

$$\sin^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right).$$

   This is asking us to find the angle, in quadrant I or IV, whose sine is $\frac{\sqrt{3}}{2}$, i.e. the answer is $\frac{\pi}{3}$.

3. (9 points) Find the exact value of $\cos(\tan^{-1}(x))$.
   Solution: Let $\theta = \tan^{-1}(x)$, so θ lies in either quadrant I or in quadrant IV. Taking tangent of both sides yields $\tan(\theta) = x$. Draw a triangle for this situation:



   Find the hypotenuse (labelled ?) by writing the Pythagorean theorem:

$$1^2 + x^2 = ?^2,$$

   and then solving for ? by taking the square root of both sides to get

$$? = \sqrt{x^2+1}.$$

Now we may compute

$$\cos(\tan^{-1}(x)) = \cos(\theta) = \frac{1}{\sqrt{x^2+1}}.$$

4. (8 points) Write the first expression in terms of the second: $\sin(-x)\csc(x)\sec(x)$; $\cos(x)$.
Solution: Compute

$$\begin{aligned}\sin(-x)\csc(x)\sec(x) &= -\sin(x)\frac{1}{\sin(x)}\frac{1}{\cos(x)} \\ &= -\frac{1}{\cos(x)}.\end{aligned}$$

5. (21 points) Establish the identity.
   (a) (7 points) $\sin(x) + \sin^3(x) + \sin(x)\cos^2(x) = 2\sin(x)$
   Solution: Start with the left:

$$\begin{aligned}\sin(x) + \sin^3(x) + \sin(x)\cos^2(x) &= \sin(x)\left[1 + \underbrace{\sin^2(x) + \cos^2(x)}_{=1}\right] \\ &= 2\sin(x)\end{aligned}$$

   (b) (7 points) $\sin(x) - 1 = \cos(x)(\tan(x) - \sec(-x))$
   Solution: Start with the right:

$$\begin{aligned}\cos(x)(\tan(x) - \sec(-x)) &= \cos(x)\left(\frac{\sin(x)}{\cos(x)} - \underbrace{\frac{1}{\cos(-x)}}_{=\frac{1}{\cos(x)}}\right) \\ &= \cos(x)\left(\frac{\sin(x)}{\cos(x)}\right) - \cos(x)\left(\frac{1}{\cos(x)}\right) \\ &= \sin(x) - 1.\end{aligned}$$

   (c) (7 points) $\dfrac{\cot(t) + \tan(t)}{\sec(-t)} = \dfrac{1}{\sin(t)}$
   Solution: Start with the left

$$\begin{aligned}\frac{\cot(t) + \tan(t)}{\sec(-t)} &= \frac{\frac{\cos(t)}{\sin(t)} + \frac{\sin(t)}{\cos(t)}}{\frac{1}{\cos(-t)}} \\ &= \frac{\frac{\cos^2(t)+\sin^2(t)}{\sin(t)\cos(t)}}{\frac{1}{\cos(t)}} \\ &= \left(\frac{1}{\sin(t)\cos(t)}\right)\left(\frac{\cos(t)}{1}\right) \\ &= \frac{1}{\sin(t)}.\end{aligned}$$

6. (8 points) Calculate, if possible. Express your answer correct to two decimal points. If this is impossible, then explain why ("my calculator said so" is not an explanation).
   (a) (2 points) $\cos^{-1}(0.837)$
   Solution: Calculate

$$\cos^{-1}(0.837) = 0.579019 \text{ rad} = 33.18°.$$

(b) (2 points) $\dfrac{1}{\cos^{-1}(1)}$

Solution: Since $\cos^{-1}(1) = 0$ we get

$$\frac{1}{\cos^{-1}(1)} = \frac{1}{0} \text{ is undefined because division by zero is not allowed.}$$

(c) (2 points) $\tan^{-1}\left(\dfrac{\pi}{3}\right)$

Solution: Calculate

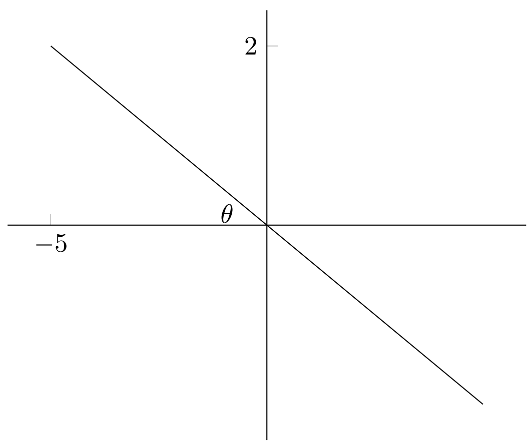
$$\tan^{-1}\left(\frac{\pi}{3}\right) = 0.8084 \text{ rad} = 46.32°.$$

(d) (2 points) $\sin^{-1}(3)$

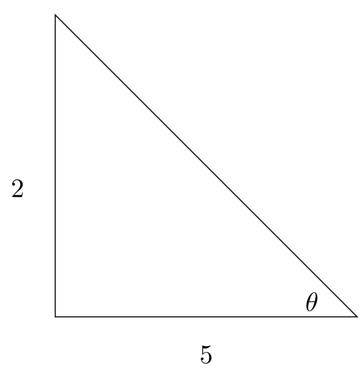
Solution: 3 is not in the domain of $\sin^{-1}$, so the original question of computing $\sin^{-1}(3)$ is not a well-posed question

7. (13 points) The line $y = -\dfrac{2}{5}x$ passes through the origin in the xy-plane. What is the angle that the line makes with the negative x-axis? Express your answer correct to two decimal places.

Solution: This function is a line through $(0, 0)$ with slope $-\dfrac{2}{5}$. Note at $x = -5$, the height of the function is 2.



We see a triangle here:

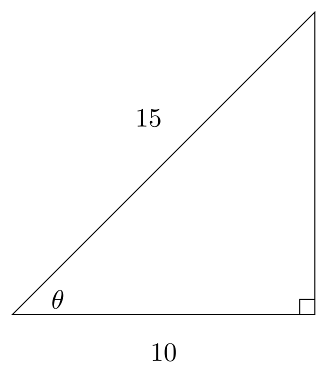


To find the angle, note that $\tan(\theta) = \dfrac{2}{5}$, and so

$$\theta = \tan^{-1}\left(\frac{2}{5}\right) \approx 0.38 \text{ rad} = 21.8°.$$

8. (13 points) A 15 foot ladder leans against a building. The bottom of the ladder is resting 10 feet from the base of the building. What angle is the ladder making with the building? Express your answer correct to two decimal points.

Solution: Draw this:



In this case, notice that $\cos(\theta) = \dfrac{10}{15}$, and so

$$\theta = \cos^{-1}\left(\frac{10}{15}\right) \approx 0.84 \text{ rad} = 48.19°.$$