

From the book, pg. 40:

**Derived Rule for Indirect Proofs R.A.A.:** *If a contradiction is derivable from a set of premises and the negation of a formula  $S$ , then  $S$  is derivable from the set of premises alone.*

In other words, to use this rule to make a conclusion  $C$ , we ...

1. Introduce  $\neg C$  as a premise,
2. Use that premise to derive a contradiction  $P \wedge \neg P$  (for any sentence  $P$  that you can find),
3. Reject  $\neg C$  as valid (it led to a contradiction), and hence
4. Conclude  $C$

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From the text, pg. 42:

“If a declaration of war is a sound strategy, then either fifty divisions are poised at the border or twenty wings of long-range bombers are ready to strike. However, fifty divisions are not poised at the border. Therefore, if twenty wings of long-range bombers are not ready to strike, then either a declaration of war is not a sound strategy or new secret weapons are available.”

Let

$D$  = “A declaration of war is a sound strategy.”

$F$  = “Fifty divisions are poised at the border.”

$T$  = “Twenty wings of long-range bombers are ready to strike.”

$S$  = “New secret weapons are available.”

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The argument has the following premises:

- {1} (1)  $D \rightarrow F \vee T$  Premise  
{2} (2)  $\neg F$  Premise

and wants to conclude the sentence  $\neg T \rightarrow (\neg D \vee S)$ . We will introduce the negation of the conclusion and then derive a contradiction from it and premises alone. In trying to do that, I will introduce the premise  $D$  on line (8) and use reductio ad absurdum to conclude  $\neg D$  and use that sentence to make the final application of reductio ad absurdum in line (15).

{1}	(1) $D \rightarrow F \vee T$	Premise
{2}	(2) $\neg F$	Premise
{3}	(3) $\neg(\neg T \rightarrow \neg D \vee S)$	Premise
{3}	(4) $\neg T \wedge \neg(\neg D \vee S)$	3 T Law of Negation for Implication
{3}	(5) $\neg T$	4 T Law of Simplification
{3}	(6) $\neg(\neg D \vee S) \wedge \neg T$	4 T Commutative law for $\wedge$
{3}	(7) $\neg(\neg D \vee S)$	6 T Law of Simplification
{8}	(8) $D$	Premise
{1, 8}	(9) $F \vee T$	1, 8 T Law of Detachment
{1, 2, 8}	(10) $T$	2, 9 T Modus tollendo ponens
{1, 2, 3, 8}	(11) $T \wedge \neg T$	5, 10 T Law of Adjunction
{1, 2, 3}	(12) $\neg D$	8, 11 R.A.A.
{1, 2, 3}	(13) $\neg D \vee S$	12 T Law of Addition
{1, 2, 3}	(14) $\neg(\neg D \vee S) \wedge (\neg D \vee S)$	7, 13 T Law of Adjunction
{1, 2}	(15) $\neg T \rightarrow \neg D \vee S$	3, 14 T R.A.A.

**#8(a) from Exam 1 (via reductio ad absurdum):** We are trying to conclude  $\neg P$  from premises (1) and (2). We will do so by admitting  $\neg(\neg P)$  as a premise and deriving a contradiction, and then applying **R.A.A.** to conclude  $\neg P$ .

{1}	(1) $P \rightarrow Q$	Premise
{2}	(2) $\neg Q$	Premise
{3}	(3) $\neg(\neg P)$	Premise
{3}	(4) $P$	3 T (Law of Double Negation)
{1, 3}	(5) $Q$	4 T (Law of Detachment)
{1, 2, 3}	(6) $Q \wedge \neg Q$	2, 5 T (Law of Adjunction)
{1, 2, }	(7) $\neg P$	3, 6 T R.A.A.

**#8(b) from Exam 1 (via reductio ad absurdum):** We are trying to prove  $S$  from using premises (1) through (4). We will do so by admitting  $\neg S$  as a premise and deriving a contraction, and then applying **R.A.A.** to conclude  $S$ .

{1}	(1) $P \rightarrow Q$	Premise
{2}	(2) $Q \rightarrow R$	Premise
{3}	(3) $S \vee (\neg R)$	Premise
{4}	(4) $P$	Premise
{5}	(5) $\neg S$	Premise
{3}	(6) $(\neg R) \vee S$	3 T (Commutativity of $\vee$ )
{3}	(7) $R \rightarrow S$	6 T (Law of Equivalence of Implication and Disjunction)
{3}	(8) $\neg S \rightarrow \neg R$	7 T (Law of Contraposition)
{3, 5}	(9) $\neg R$	5, 8 T (Law of Detachment)
{1}	(10) $\neg Q \rightarrow \neg P$	1 T (Law of Contraposition)
{2}	(11) $\neg R \rightarrow \neg Q$	2 T (Law of Contraposition)
{2, 3, 5}	(12) $\neg Q$	9, 11 T (Law of Detachment)
{1, 2, 3, 5}	(13) $\neg P$	10, 12 T (Law of Detachment)
{1, 2, 3, 4, 5}	(14) $P \wedge (\neg P)$	4, 13 T (Law of Adjunction)
{1, 2, 3, 4}	(15) $S$	5, 14 T R.A.A.