

From the text, pg. 29:

Rule C.P.: *If we can derive S from R and a set of premises, then we may derive $R \rightarrow S$ from the premises alone.*

Question from 14 February class: when Rule C.P. is invoked and when R is a premise, do we list the line corresponding to R when invoking rule C.P.?

Answer: The answer is no, see examples of this on line (8) of the deduction on pg. 29 and line (9) in the example on pg. 30. The key fact about **Rule C.P.** is that it is the only rule we have at this point that *allows you to remove premises from the left column*. Practically, this means, in some sense, that we may introduce a new premise, use it to “do something”, and then make it “go away”.

A “practical” use of Rule C.P.: can you deduce $P \rightarrow R$ from the following premises?:

{1} (1) $P \rightarrow Q$ Premise

{2} (2) $\neg(\neg Q) \rightarrow R$ Premise

It seems “obvious” that we should be able to conclude $P \rightarrow R$ from this, but it also seems difficult to deduce formally because the sentence $Q \rightarrow S$ does *not* follow from the Law of Double Negation (it doesn’t match the “correct form”). This is an instance where exploiting **Rule P** along with **Rule C.P.** is the easiest way to proceed. Recall what **Rule P** says (pg. 28):

Rule P: *We may introduce a premise at any point of the derivation.*

The strategy: We will introduce a new premise Q and then use it to conclude $\neg\neg Q$, from which we may deduce R . This shows that “*we can derive R from Q and a set of premises*”, and so **Rule C.P.** tells us that “*we may derive $Q \rightarrow R$ from the premises alone*”

{1} (1) $P \rightarrow Q$ Premise

{2} (2) $\neg(\neg Q) \rightarrow R$ Premise

{3} (3) Q Premise

{3} (4) $\neg\neg Q$ 3 T (Law of Double Negation)

{2, 3} (5) R 2,4 T (Law of Detachment)

{2} (6) $Q \rightarrow R$ 3, 5 C.P.

{1, 2} (7) $P \rightarrow R$ 1,6 T (Law of Hypothetical Syllogism)

Challenge: Can you deduce $P \rightarrow R$ from premises (1) and (2) without using **Rule C.P.**?

Additional comment: I just want to point out again that there are logics out there which do not accept the “Law of Double Negation” – [intuitionistic logic](#) is probably the most common (read the “syntax” section on its wikipedia page!)