Homework 12

1. Recall the definition of the Ack function in the 20 April slides. Compute $\mathrm{Ack}(1, 2)$.
2. Consider a function defined recursively by

$$x(n+1) = (n+1) * x(n), \quad x(0) = 3.$$

What is $x(4)$? What is $x(5)$? In general, what is the (non-recursive) formula for $x(n)$ (for $n = 0, 1, 2, 3, \ldots$)?

Consider the theory of four-point geometry, defined by the following axioms:

Axiom 1	There exist exactly four points.
Axiom 2	Each two distinct points have exactly one line that contains both of them.
Axiom 3	Each line is on exactly two points.

3. Draw a picture (i.e. "model") of four-point geometry.
4. Show that Axiom 1 of four-point geometry is independent of axioms 2 and 3.
5. Show that Axiom 2 of four-point geometry is independent of axioms 1 and 3.
6. Show that Axiom 3 of four-point geometry is independent of axioms 1 and 2.

Fano's geometry is defined by the following five axioms:

Axiom F1	There exists at least one line.
Axiom F2	Every line has exactly three points on it.
Axiom F3	Not all the points are on the same line.
Axiom F4	For two distinct points, there exists exactly one line on both of them.
Axiom F5	Each two lines have at least one point on both of them.

7. Draw a model for Fano geometry. (hint: letting your "lines" be curved can help make this easier)

In the solution for homework 10, the following sequence of formulas is provided to prove that the formula $S0 \cdot SS0 = S0 + S0$ is a theorem of first order arithmetic (this is problem 2b):

Formulas used in proof of HW10 problem 2b
$S0 \cdot S0 = S0$
$S0 \cdot SS0 = (S0 \cdot S0) + S0$
$S0 \cdot SS0 = S0 + S0$

Consider the following assignment of values for symbols:

Symbol in the theory	Assigned numerical value
S	2
0	3
$\cdot$	4
$=$	5
(	6
)	7
$+$	8

8. Find the Gödel number of the formula in each line.
9. Find the super Gödel number of the proof (from the solution) of problem 2b in HW10.