Homework 7 - MATH 2200 Spring 2017

1. Formula or not? If not, circle the problem.

(a) $(\forall x)(\exists y)(Px \wedge (Gy \to (\exists z)(Qz)))$
Solution: This is a formula.

(b) $(\forall w)(\exists \textcircled{P} x)(Gpx) \vee (\forall x)(Wx \vee \neg Wx)$
Solution: The circled part is a problem because capital letters are reserved for predicates. Notice that our rules for formulas, on pg. 52 (d), says that the only way a quantifier like $\forall$ and $\exists$ may be part of a formula is if it is next to a *variable* – it does not allow quantification over predicates.
note: this restriction is imposed because we are formalizing "first-order logic" instead of "second-order logic"

(c) $(\exists a)(\exists b)(\forall w)(Qw \leftrightarrow (Pab \to Gbw)) \wedge Px$
Solution: This is a formula. It may appear strange since the variable x is free, but formulas are allowed to have free variables. *Sentences* are formulas that cannot have free variables (pg. 54 in the text).

(d) $(\forall \textcircled{\exists})(G\textcircled{\exists} \wedge P\textcircled{\forall})$
Solution: The first error is an error because the symbol $\exists$ is reserved for the existential quantifier, and it never stands for a variable, which is what must always follow any $\forall$. The second error is because it appears as if the predicate G is acting on the "variable" $\exists$ – this is an error similar to the first. The third error is similar: $\forall$ is being used as if it was a variable, while it is never one.

2. Write a formal deduction for the argument:

"All norms yield a metric. All things that yield a metric also yield a topology. Therefore all norms yield a topology."

Solution: We read this semi-formally as

"For all x, if x is a norm, then x yields a metric. For all x, if x yields a metric, then x yields a topology. Therefore for all x, if x is a norm, then x yields a topology."

We will let N be the predicate "is a norm", M be the predicate "yields a metric", and T be the predicate "yields a topology". First note our premises:

{1}	(1) $(\forall x)(Nx \to Mx)$	Premise
{2}	(2) $(\forall x)(Mx \to Tx)$	Premise

We would like to conclude $(\forall x)(Mx \to Tx)$:

{1}	(1) $(\forall x)(Nx \to Mx)$	Premise
{2}	(2) $(\forall x)(Mx \to Tx)$	Premise
{1}	(3) $Nx \to Mx$	1 US
{2}	(4) $Mx \to Tx$	2 US
{1, 2}	(5) $Nx \to Tx$	1 2 T Law of Hypothetical Syllogism
{1, 2}	(6) $(\forall x)(Nx \to Tx)$	5 UG

3. Write a formal deduction for the argument:

"All vector spaces are a group under addition. The set of positive integers do not form a group under addition. Therefore the set of positive integers is not a vector space."

Solution: We read this semi-formally, letting z stand in for the "set of positive integers" as

"For all x, if x is a vector space, then x is a group under addition. z is not a group under addition. Therefore, z is not a vector space."

We let V be the predicate "is a vector space" and G be the predicate "is a group under addition". Our premises are

{1}	(1) $(\forall x)(Vx \to Gx)$	Premise
{2}	(2) $\neg Gz$	Premise

We would like to conclude $\neg Vz$:

{1}	(1) $(\forall x)(Vx \to Gx)$	Premise
{2}	(2) $\neg Gz$	Premise
{1}	(3) $Vz \to Gz$	1 US
{1}	(4) $\neg Gz \to \neg Vz$	3 T Law of Contraposition
{1, 2}	(5) $\neg Vz$	2 3 T Law of Detachment