Homework 3 - MATH 2200 Spring 2017

1. Construct a counterexample to show that the following rule of inference is invalid: from $\neg P$ and $P \rightarrow Q$, we may derive $\neg Q$.

2. There is an error in the following deduction. Find it and explain why it is an error. Recall that the **Law of Contraposition** is the following tautological implication: $(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$ and the **Law of Detachment** says the following is a tautological implication: $[P \wedge (P \rightarrow Q)] \rightarrow Q$.

$\{1\}$	$(1) P \rightarrow Q$	Premise
$\{2\}$	$(2) Q \rightarrow R$	Premise
$\{3\}$	$(3) \neg R$	Premise
$\{2\}$	$(4) \neg R \rightarrow \neg Q$	2 T (Law of Contraposition)
$\{2, 3\}$	$(5) \neg Q$	3,4 T (Law of Detachment)
$\{1, 2, 3\}$	$(6) P$	1, 5 T (Law of Detachment)

3. Is the following formal argument valid? Recall that the **Law of Hypothetical Syllogism** says that $[(P \rightarrow Q) \wedge (Q \rightarrow R)] \rightarrow (P \rightarrow R)$ is a tautological implication. Let P denote the sentence "$1 + 1 = 3$", let Q denote "$1 = 2$", and let R denote "$0 = 1$".

$\{1\}$	$(1) P \rightarrow Q$	Premise
$\{2\}$	$(2) Q \rightarrow R$	Premise
$\{3\}$	$(3) P$	Premise
$\{1, 2\}$	$(4) P \rightarrow R$	1,2 T (Law of Hypothetical Syllogism)
$\{1, 2, 3\}$	$(5) R$	3,4 T (Law of Detachment)

4. Construct a formal deduction for the given argument. Do so by applying the **Law of Absurdity**, which says that $[(P \rightarrow Q) \wedge \neg Q] \rightarrow \neg P$:
"If the Thomae function is differentiable, then the Thomae function is continuous. The Thomae function is not continuous. Therefore the Thomae function is not differentiable."

5. Construct a formal deduction for the given argument. Do so by applying the **Law of Detachment**:
"If the function f is entire, then the contour integral of f over the circle is zero. The function f is entire. Therefore the contour integral of f over the circle is zero."