Homework 10

1. Prove the following in pure identity theory or give an interpretation to show it is not true:

a.) $(x = y \wedge (y = z \wedge z = w)) \rightarrow x = w$

b.) $\neg(x = y) \wedge \neg(y = z) \rightarrow \neg(x = z)$

2. Prove the following in first order arithmetic or give an interpretation to show it is not true:

a.) $S0 \cdot S0 = S0$

b.) $S0 \cdot SS0 = S0 + S0$

c.) $SS0 \cdot SSS0 = SSSS0$

3. Consider a new theory called "total order theory" which has a two-place predicate "$\leq$" and the following axioms:

Axiom 1	$(\forall x)(\forall y)(\forall z)(x \leq y \wedge y \leq z \rightarrow x \leq z)$
Axiom 2	$(\forall x)(x \leq x)$
Axiom 3	$(\forall x)(\forall y)(x \leq y \wedge y \leq x \rightarrow x = y$
Axiom 4	$(\forall x)(\forall y)(x \leq y \vee y \leq x)$

Prove the following in total order theory or give an interpretation to show it is not true:

a.) $(\forall x)(\forall y)(\neg(x \leq y) \rightarrow y \leq x)$

b.) $x \leq y \wedge \neg(x \leq z) \rightarrow \neg(y \leq z)$

c.) $\neg(x \leq y) \wedge \neg(y \leq z) \rightarrow \neg(x \leq z)$