MATH 1540 - EXAM 1 - FALL 2017
SOLUTION

Name: _____

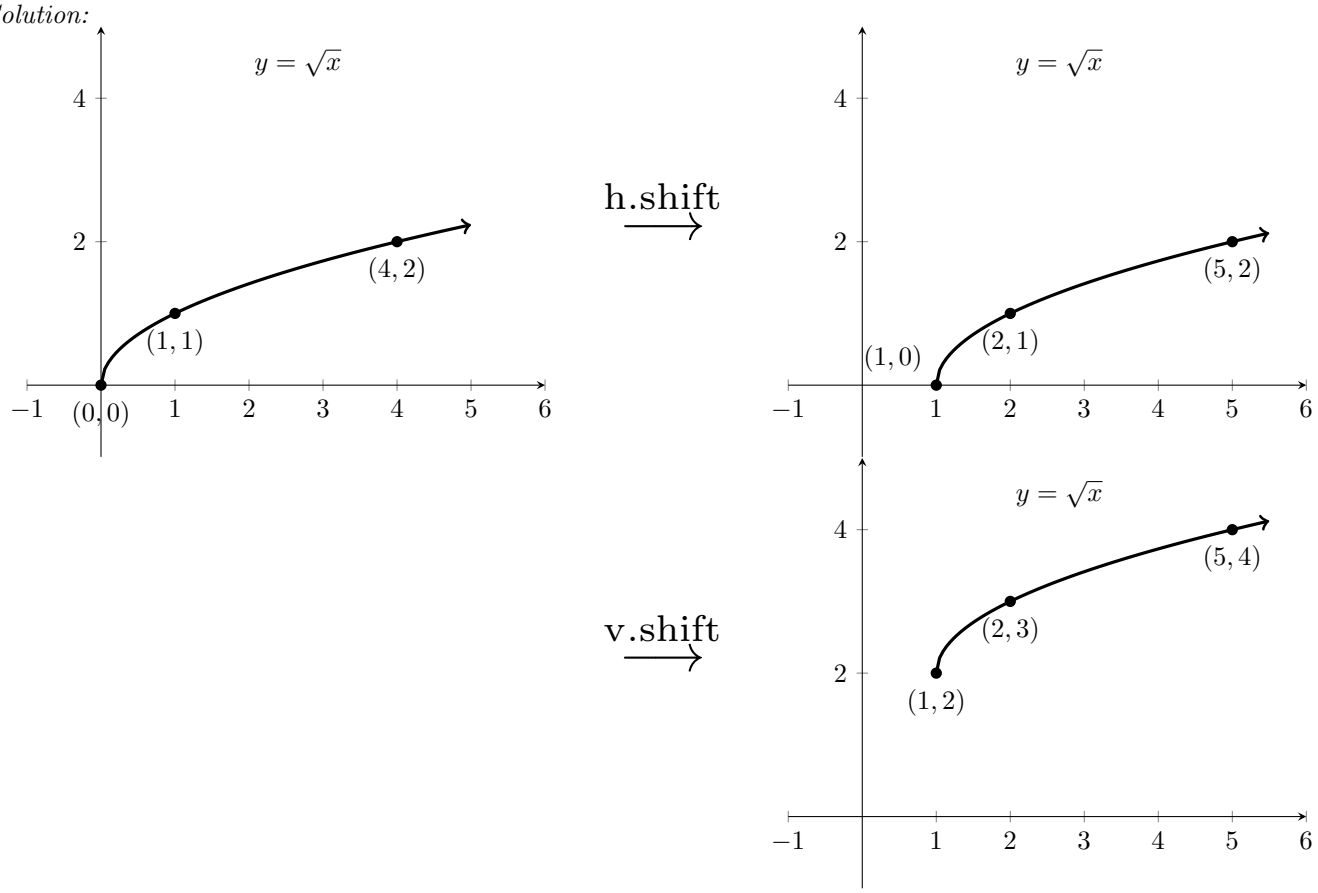
Thursday 7/Friday 8 September 2017
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (5 points) Sketch a graph of the equation $y = 2 + \sqrt{x-1}$. Include at last two labeled points in your plot to guarantee full credit.

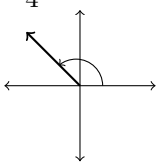
Solution:



2. (6 points) Draw the specified angle.

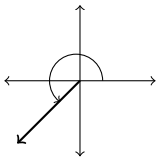
   (a) (2 points) $\frac{3\pi}{4}$ radians

   Solution:



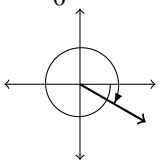
   (b) (2 points) $237°$

   Solution:

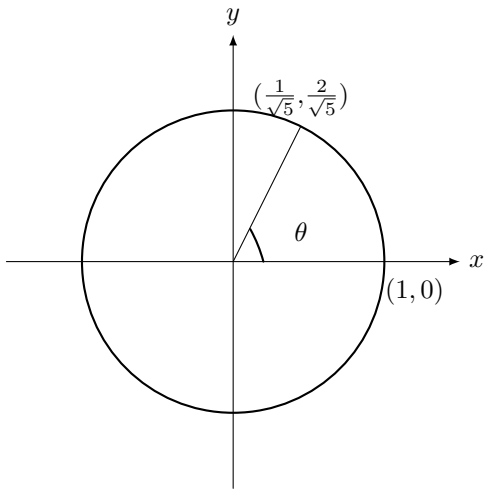


   (c) (2 points) $-\frac{13\pi}{6}$ radians

   Solution:



3. (6 points) Here is a portion of the unit circle, partially labeled:



(a) (2 points) What is $\cos(\theta)$?

Solution: $\dfrac{1}{\sqrt{5}}$

(b) (2 points) What is $\cot(\theta)$?

Solution: $\dfrac{1/\sqrt{5}}{2/\sqrt{5}} = \dfrac{1}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{2} = \dfrac{1}{2}$

(c) (2 points) What is $\csc(\theta)$?

Solution: $\dfrac{1}{2/\sqrt{5}} = \dfrac{\sqrt{5}}{2}$

4. (12 points) (a) (4 points) Convert $119°$ into radians.

   Solution: $119° = 119° \cdot \dfrac{\pi}{180°} = \dfrac{119\pi}{180}$

   (b) (4 points) Convert $\dfrac{7\pi}{13}$ radians into degrees.

   Solution: $\dfrac{7\pi}{13} = \dfrac{7\pi}{13} \cdot \dfrac{180°}{\pi} = \dfrac{7 \cdot 180}{13} = \dfrac{1260}{13}$

   (c) (4 points) Convert 5 radians to degrees.

   Solution: $5 = 5 \cdot \dfrac{180}{\pi} = \dfrac{900}{\pi}$

5. (4 points) What quadrant is θ in if...

   (a) (2 points) $\cos(\theta) > 0$ and $\sin(\theta) < 0$.

   Solution: IV.

   (b) (2 points) $\tan(\theta) < 0$ and $\csc(\theta) > 0$

   Solution: II

6. (12 points) Find an exact value for...

   (a) (4 points) $\sin\left(\dfrac{\pi}{2}\right)$

   Solution: $\sin\left(\dfrac{\pi}{2}\right) = 1$

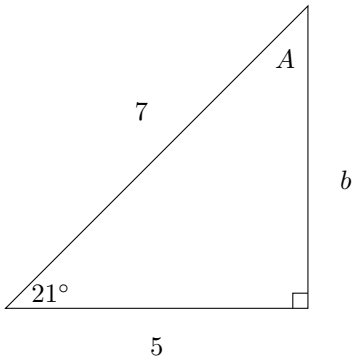
   (b) (4 points) $\cot\left(\dfrac{\pi}{3}\right)$

   Solution: $\cot\left(\dfrac{\pi}{3}\right) = \dfrac{1/2}{\sqrt{3}/2} = \dfrac{1}{\sqrt{3}}$

   (c) (4 points) $\cos\left(\dfrac{11\pi}{6}\right)$

   Solution: $\cos\left(\dfrac{11\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

7. (12 points) (a) (6 points) Solve the triangle.



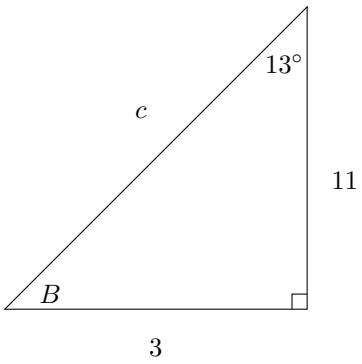
Solution:
$A = 180° - 90° - 21° = \boxed{69°}$

$25 + b^2 = 49$

$b^2 = 24$

$\boxed{b = \sqrt{24}}$

(b) (6 points) Solve the triangle.



Solution:
$B = 180° - 90° - 13° = \boxed{77°}$

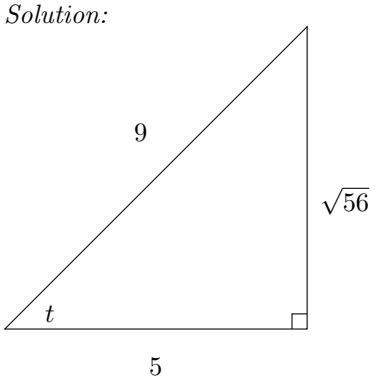
$3^2 + 11^2 = c^2$

$130 = c^2$

$\boxed{c = \sqrt{130}}$

8. (15 points) If $\cos(t) = \dfrac{5}{9}$ and t is in quadrant IV, find the other five trigonometric functions.

Solution:



$$\sin(t) = -\frac{\sqrt{56}}{9} \quad \csc(t) = -\frac{9}{\sqrt{56}}$$
$$\cos(t) = \frac{5}{9} \quad \sec(t) = \frac{9}{5}$$
$$\tan(t) = -\frac{\sqrt{56}}{5} \quad \cot(t) = -\frac{5}{\sqrt{56}}$$

$a^2 = 9^2 - 5^2 = 81 - 25 = 56 \to a = \sqrt{56}$

9. (8 points) Find the arc length subtended by an angle of $\theta = 37°$ of a circle of radius 3. Express your final answer accurate to at least two decimal places.

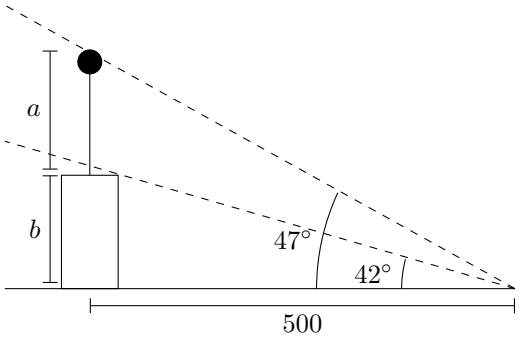
Solution:
First, convert θ to radians:

$$\theta = 37° = 37° \cdot \frac{\pi}{180°} = \frac{37\pi}{180}.$$

$$s = r\theta$$
$$s = 3 \cdot \frac{37\pi}{180}$$
$$s = \frac{111\pi}{180}$$
$$\boxed{s = 1.9373}$$

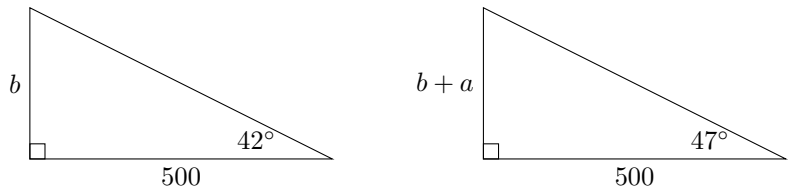
10. (16 points) There is an antenna on the top of a building. From a location 500 feet from the base of the building, the angle of elevation to the top of the building is measured to be 42°. From the same location, the angle of elevation to the top of the antenna is measured to be 47°. Find the height of the antenna. Express your final answer accurate to at least two decimal places.

Solution: We start with a figure to describe the problem:



Thus, we seek a.

Note that we have *two* right triangles in this picture, reproduced (in a simplified form) below:



Thus,

$$\tan(42^\circ) = \frac{b}{500}$$
$$b = 500\tan(42^\circ)$$

and

$$\tan(47^\circ) = \frac{b+a}{500}$$
$$b + a = 500\tan(47^\circ)$$

So that

$$a = (b+a) - b$$
$$= 500\tan(47^\circ) - 500\tan(42^\circ)$$

$\boxed{a \approx 85.98 \text{ feet.}}$

11. (4 points) Use your calculator to compute the following trig functions accurate to at least two decimal places.

(a) (1 point) $\tan(21^\circ)$

Solution:
$\tan(21^\circ) = 0.3838$

(b) (1 point) $\cos\left(\frac{8\pi}{13}\right)$

Solution:
$\cos\left(\frac{8\pi}{13}\right) = -0.3546$

(c) (1 point) $\csc(57^\circ)$

Solution:
$\csc(57^\circ) = 1.192$

(d) (1 point) $\tan(21)$

Solution:
$\tan(21) = -1.527$