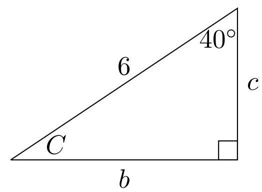
HW 12:

(1) Solve the right triangle $a = 6, B = 40°$.
Solution: First draw the triangle:



Find C

Recall that the right angle is 90°. Use the fact that the sum of the angles of a triangle is 180° to see

$$C + 40° + 90° = 180°,$$

giving us

$$C + 130° = 180°,$$

so

$$C = 180° - 130° = 50°.$$

Find b

We know the angle $B = 40°$ and the hypotenuse, so since the sine function obeys $\sin(\theta) = \frac{\text{opposite of } \theta}{\text{hypotenuse}}$, we get

$$\sin(40°) = \frac{b}{6},$$

and so

$$b = 6\sin(40°).$$

Find c

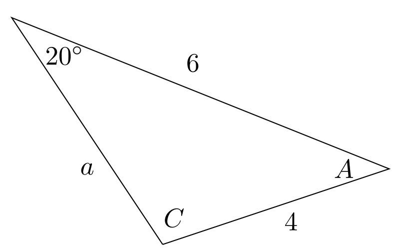
We know $B = 40°$ and the hypotenuse, so since the cosine function obeys $\cos(\theta) = \frac{\text{adjacent to } \theta}{\text{hypotenuse}}$, we see

$$\cos(40°) = \frac{c}{6},$$

so

$$c = 6\cos(40°).$$

(2) Solve the triangle given by the information $b = 4$, $c = 6$, and $B = 20°$.
Solution: First draw what is described:



The only choice we have here is to use the law of sines. If one tries the law of cosines it always yields two variables. We want to use the pair 20° and the 4 on one side and we are forced to use the angle C and side 6.

Find C

Using the law of sines,

$$\frac{\sin(20°)}{4} = \frac{\sin(C)}{6},$$

so

$$\frac{6\sin(20°)}{4} = \sin(C),$$

therefore

$$C = \sin^{-1}\left(\frac{6\sin(20°)}{4}\right) \overset{\text{table}}{=} 30.8°.$$

We also have to check the other possible angle:

$$180° - 30.8° = 149.2°,$$

and this angle is possible – it is not too big. Therefore we will have to solve two resulting triangles.

Find A

Now that we know two of the three angles, it is easy to find the third angle:

$$20° + 30.8° + A = 180°,$$

so

$$A + 50.8° = 180°.$$

Therefore

$$A = 180° - 50.8° = 129.2°.$$

Find a

Now we use the law of sines again to see

$$\frac{\sin(129.2°)}{a} = \frac{\sin(20°)}{4}.$$

Therefore

$$a = \frac{4\sin(129.2°)}{\sin(20°)}.$$

The law of cosines *could* be used to solve for a:

$$a^2 = 4^2 + 6^2 - 2(4)(6)\cos(129.2°),$$

so

$$a^2 = 16 + 36 - 48\cos(129.2°),$$

hence

$$a = \sqrt{52 - 48\cos(129.2°)}.$$

Find A

Now that we know two of the three angles, it is easy to find the third angle:

$$20° + 149.2° + A = 180°,$$

so

$$A + 169.2° = 180°.$$

Therefore

$$A = 180° - 169.2° = 10.8°.$$

Find a

Now we use the law of sines again to see

$$\frac{\sin(10.8°)}{a} = \frac{\sin(20°)}{4}.$$

Therefore

$$a = \frac{4\sin(10.8°)}{\sin(20°)}.$$

The law of cosines *could* be used to solve for a:

$$a^2 = 4^2 + 6^2 - 2(4)(6)\cos(10.8°),$$

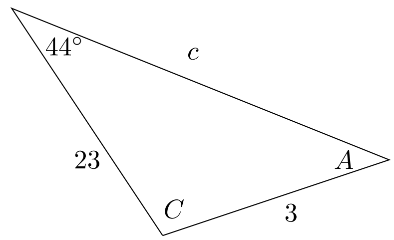
so

$$a^2 = 16 + 36 - 48\cos(10.8°),$$

hence

$$a = \sqrt{52 - 48\cos(10.8°)}.$$

(3) Solve the triangle given by the information $a = 23$, $b = 3$, and $B = 44°$.
Solution: First draw what is described:



We are forced to use the law of sines to find the angle A.
Find A
Using the law of sines,

$$\frac{\sin(44°)}{3} = \frac{\sin(A)}{23}.$$

Therefore

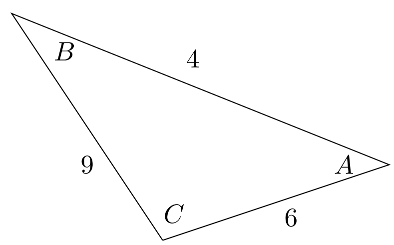
$$\frac{23\sin(44°)}{3} = \sin(A),$$

and so

$$A = \sin^{-1}\left(\frac{23\sin(44°)}{3}\right),$$

but this does not exist (from the table). Therefore there is no triangle to complete the solution for.

(4) Solve the triangle given by the information $a = 9$, $b = 6$, and $c = 4$.
Solution: First draw what is described:



We are forced to use the law of cosines to find an angle (so we will have to use it twice to find two angles).
Find A
Using the law of cosines,

$$9^2 = 4^2 + 6^2 - 2(4)(6)\cos(A)$$

Therefore

$$81 = 52 - 48\cos(A).$$

Therefore

$$29 = -48\cos(A),$$

hence

$$\frac{29}{-48} = \cos(A).$$

Take $\cos^{-1}$ of both sides and we get

$$A = \cos^{-1}\left(-\frac{29}{48}\right) \overset{\text{table}}{=} 127.2°.$$

<u>Find B</u>
Using the law of cosines,

$$6^2 = 9^2 + 4^2 - 2(9)(4)\cos(B),$$

so

$$36 = 81 + 16 - 72\cos(B).$$

Simplifying,

$$36 = 97 - 72\cos(B),$$

subtract 97 to get

$$-61 = -72\cos(B),$$

therefore

$$\frac{-61}{-72} = \cos(B),$$

and we get

$$B = \cos^{-1}\left(\frac{61}{72}\right) = 32.1°.$$

<u>Find C</u>
Now that we have A and B, we use the sum of the angles of a triangle being 180° to get

$$127.2° + 32.1° + C = 180°,$$

and so

$$159.3° + C = 180°.$$

Therefore

$$C = 180° - 159.3° = 20.7°.$$

Also note we could have also found C using the law of cosines:

$$4^2 = 9^2 + 6^2 - 2(9)(6)\cos(C),$$

yielding

$$16 = 117 - 108\cos(C),$$

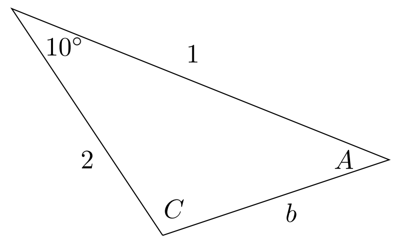
so

$$-101 = -108\cos(C),$$

hence

$$C = \cos^{-1}\left(\frac{101}{108}\right) \overset{\text{table}}{=} 20.7°.$$

(5) Solve the triangle given by the information $a = 2$, $B = 10°$, $c = 1$.
Solution: First draw what is described:



We are forced to use the law of cosines to first find side b.
<u>Find b</u>
Using the law of cosines,

$$b^2 = 2^2 + 1^2 - 2(2)(1)\cos(10°).$$

Therefore

$$c = \sqrt{5 - 4\cos(10°)} \overset{\text{table}}{=} 1.03.$$

<u>Find C</u>
To find C, use the law of cosines:

$$1^2 = 2^2 + 1.03^2 - 2(2)(1.03)\cos(C),$$

so

$$1 = 4 + 1.06 - 4.12\cos(C).$$

Simplifying,

$$1 = 5.06 - 4.12\cos(C),$$

so

$$-4.06 = -4.12\cos(C),$$

hence

$$C = \cos^{-1}\left(\frac{4.06}{4.12}\right) = 9.7°$$

<u>Find A</u>
Since the sum of the angles of a triangle is 180° we see

$$10° + 9.7° + A = 180°,$$

so

$$19.7° + A = 180°,$$

yielding

$$A = 180° - 19.7° = 160.3°.$$

If instead the law of sines is used to find A:

$$\frac{\sin(A)}{2} = \frac{\sin(10°)}{1.03},$$

so

$$A = \sin^{-1}\left(\frac{2\sin(10°)}{1.03}\right) \overset{\text{table}}{=} 19.7°.$$

Taking $a = 19.7°$ forces $C = 180° - 10° - 19.7° = 150.3$.
We also have to check the angle

$$180° - 19.7° = 160.3°$$

This would mean that $C = 180° - 10° - 160.3° = 9.7°$. However if the law of sines is used to find C instead, we would find

$$\frac{\sin(C)}{1} = \frac{\sin(10°)}{1.03},$$

so

$$C = \sin^{-1}\left(\frac{\sin(10°)}{1.03}\right) \overset{\text{table}}{=} 9.7°.$$

This means that $A = 180° - 10° - 9.7° = 160.3°$.
We also have to check the angle

$$180° - 9.7° = 170.3°$$

This would mean that

$$C = 180° - 10° - 170.3° = -0.3$$

which is nonsense!
Therefore the solution must be $C = 9.7°$ and $A = 160.3°$.

$$\sin^{-1}\left(\frac{6\sin(20^\circ)}{4}\right) = 30.8^\circ$$
$$\sin^{-1}\left(\frac{6}{4\sin(20^\circ)}\right) \textbf{ DNE}$$
$$\sin^{-1}\left(\frac{3\sin(44^\circ)}{23}\right) = 5.2^\circ$$
$$\sin^{-1}\left(\frac{3}{23\sin(44^\circ)}\right) = 10.8^\circ$$
$$\cos^{-1}\left(\frac{61}{72}\right) = 32.1^\circ$$
$$\cos^{-1}\left(-\frac{61}{72}\right) = 147.9^\circ$$
$$\cos^{-1}\left(\frac{29}{48}\right) = 52.8^\circ$$
$$\cos^{-1}\left(-\frac{29}{48}\right) = 127.2^\circ$$
$$\cos^{-1}\left(\frac{101}{108}\right) = 20.7^\circ$$
$$\cos^{-1}\left(-\frac{101}{108}\right) = 159.3^\circ$$
$$\sqrt{5-4\cos(10^\circ)} = 1.03$$
$$\cos^{-1}\left(\frac{1-4-1.03^2}{-2(2)(1.03)}\right) = \cos^{-1}\left(\frac{4.06}{4.12}\right) = 9.79^\circ$$
$$\cos^{-1}\left(\frac{1-4-1.03^2}{2(2)(1.03)}\right) = \cos^{-1}\left(-\frac{4.06}{4.12}\right) = 170.2^\circ$$
$$\cos^{-1}\left(\frac{1-4-2.08^2}{-2(2)(2.08)}\right) = \cos^{-1}\left(\frac{7.33}{8.32}\right) = 28.24^\circ$$
$$\cos^{-1}\left(\frac{1-4-2.08^2}{2(2)(2.08)}\right) = \cos^{-1}\left(-\frac{7.33}{8.32}\right) = 151.76^\circ$$
$$\cos^{-1}\left(\frac{4-1.03^2-1}{-2(1.03)}\right) = \cos^{-1}\left(-\frac{1.94}{2.06}\right) = 160.35^\circ$$
$$\cos^{-1}\left(\frac{4-1.03^2-1}{2(1.03)}\right) = \cos^{-1}\left(\frac{1.94}{2.06}\right) = 19.65^\circ$$
$$\sin^{-1}\left(\frac{2\sin(10^\circ)}{1.03}\right) = 19.71^\circ$$
$$\sin^{-1}\left(\frac{\sin(10^\circ)}{1.03}\right) = 9.71^\circ$$

$$\sin^{-1}\left(\frac{4\sin(20^\circ)}{6}\right) \textbf{ DNE}$$
$$\sin^{-1}\left(\frac{4}{6\sin(20^\circ)}\right) \textbf{ DNE}$$
$$\sin^{-1}\left(\frac{23\sin(44^\circ)}{3}\right) \textbf{ DNE}$$
$$\sin^{-1}\left(\frac{23}{3\sin(44^\circ)}\right) \textbf{ DNE}$$
$$\cos^{-1}\left(\frac{72}{61}\right) \textbf{ DNE}$$
$$\cos^{-1}\left(-\frac{72}{61}\right) \textbf{ DNE}$$
$$\cos^{-1}\left(\frac{48}{29}\right) \textbf{ DNE}$$
$$\cos^{-1}\left(-\frac{48}{29}\right) \textbf{ DNE}$$
$$\cos^{-1}\left(\frac{108}{101}\right) \textbf{ DNE}$$
$$\cos^{-1}\left(-\frac{108}{101}\right) \textbf{ DNE}$$
$$\sqrt{5-4\sin(10^\circ)} = 2.08$$
$$\cos^{-1}\left(\frac{4.12}{4.06}\right) \textbf{ DNE}$$
$$\cos^{-1}\left(-\frac{4.12}{4.06}\right) \textbf{ DNE}$$
$$\cos^{-1}\left(\frac{8.32}{7.33}\right) \textbf{ DNE}$$
$$\cos^{-1}\left(-\frac{8.32}{7.33}\right) \textbf{ DNE}$$
$$\cos^{-1}\left(\frac{-2.06}{1.94}\right) \textbf{ DNE}$$
$$\cos^{-1}\left(\frac{2.06}{1.94}\right) \textbf{ DNE}$$
$$\sin^{-1}\left(\frac{2\sin(10^\circ)}{2.08}\right) = 9.61^\circ$$
$$\sin^{-1}\left(\frac{\sin(10^\circ)}{2.08}\right) = 4.79^\circ$$