MATH 1115 - EXAM 3 FALL 2016
SOLUTION

Thursday 17 November 2016
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (15 points) Solve the following trigonometric equations as specified.

(a) (5 points) Find the general solution of $\sin(\theta) = \frac{1}{2}$.
Solution: From the unit circle, the solution is

$$\begin{cases} \theta = \frac{\pi}{6} + 2\pi k \\ \theta = \frac{5\pi}{6} + 2\pi k. \end{cases}$$

(b) (5 points) Solve $4\cos^2(\theta) = 1$ for $0 \leq \theta \leq 2\pi$.
Solution: First isolate $\cos^2(\theta)$ to get

$$\cos^2(\theta) = \frac{1}{4}.$$

Now take the square root of both sides yielding

$$\cos(\theta) = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}.$$

From the unit circle, we see that the solution is

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}.$$

(c) (5 points) Solve $\tan(2\theta) = 1$ for $0 \leq \theta \leq 2\pi$.
Solution: By the unit circle, the general solution of $\tan(2\theta) = 1$ is

$$\begin{cases} 2\theta = \frac{\pi}{4} + 2\pi k \\ 2\theta = \frac{5\pi}{4} + 2\pi k. \end{cases}$$

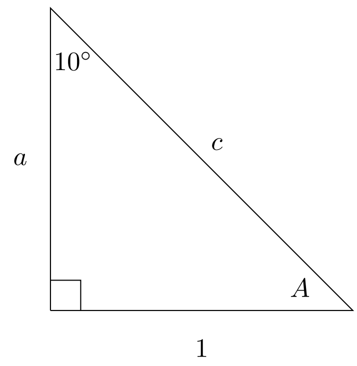
Solving each for θ by dividing by 2 yields

$$\begin{cases} \theta = \frac{\pi}{8} + \pi k \\ \theta = \frac{5\pi}{8} + \pi k. \end{cases}$$

For which k do we get values of θ between 0 and 2π? Some calculation shows that we can take $k = 0$ and $k = 1$ yielding four total solutions

$$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}.$$

2. (15 points) Solve the following right triangle:



Solution: **<u>Find A</u>**
Using the fact that sum of the angles in a triangle is 180° we get

$$10^\circ + 90^\circ + A = 180^\circ.$$

Therefore

$$A = 80^\circ.$$

<u>Find a</u>
There are a number of ways to do this. We will proceed by noting that

$$\tan(10^\circ) = \frac{1}{a},$$

and so

$$a = \frac{1}{\tan(10^\circ)}.$$

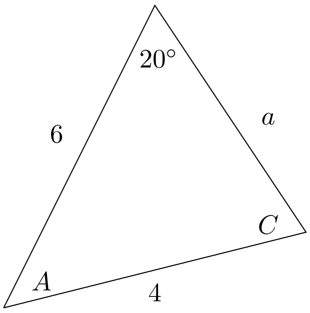
<u>Find c</u> There are a number of ways to do this. We will proceed by noting that

$$\sin(10^\circ) = \frac{1}{c},$$

so

$$c = \frac{1}{\sin(10^\circ)}.$$

3. (15 points) Solve the following triangle:



Solution: **Find C**

Using the law of sines,

$$\frac{\sin(20^\circ)}{4} = \frac{\sin(C)}{6}.$$

Isolate $\sin(C)$ to get

$$\frac{6\sin(20^\circ)}{4} = \sin(C).$$

Now solve for C by taking $\sin^{-1}$ of both sides to get

$$C = \sin^{-1}\left(\frac{6\sin(20^\circ)}{4}\right) \stackrel{\text{table}}{=} 30.9^\circ.$$

Be careful! You must also check to see if there is a second solution. The other possibility would be for

$$C = 180^\circ - 30.9^\circ = 149.1^\circ.$$

This angle is not too big, so we must solve a second triangle associated with this value for C. We will do that solution in blue after we do the solution for the first value of C.

Find A

Using the fact that the sum of the angles of a triangle is 180° to write

$$20^\circ + 30.9^\circ + A = 180^\circ.$$

This yields

$$A = 129.1^\circ.$$

Proceed similarly using the other value for C:

$$20^\circ + 149.1^\circ + A = 180^\circ.$$

Solving for A we get

$$A = 10.9^\circ.$$

Find a

Using the law of sines (you could also use the law of cosines),

$$\frac{\sin(129.1^\circ)}{a} = \frac{\sin(20^\circ)}{4}.$$

Therefore

$$a = \frac{4\sin(129.1^\circ)}{\sin(20^\circ)}.$$

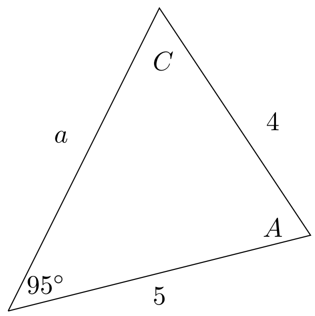
In the second triangle, use the law of sines to find a:

$$\frac{\sin(10.9^\circ)}{a} = \frac{\sin(20^\circ)}{4}$$

and solving for a yields

$$a = \frac{4\sin(10.9^\circ)}{\sin(20^\circ)}.$$

4. (15 points) Solve the following triangle:



Solution: **<u>Find C</u>**
Using the law of sines,

$$\frac{\sin(C)}{5} = \frac{\sin(95°)}{4}.$$

Isolate $\sin(C)$ to get

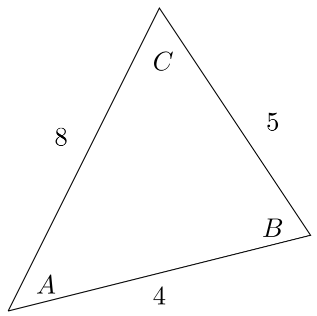
$$\sin(C) = \frac{5\sin(95°)}{4}.$$

Now take $\sin^{-1}$ of both sides to get

$$C = \sin^{-1}\left(\frac{5\sin(95°)}{4}\right),$$

BUT from the table, we see that this quantity **does not exist**. Therefore this triangle has no solution.

5. (15 points) Solve the following triangle:



Find A
Using the law of cosines,

$$5^2 = 8^2 + 4^2 - 2(8)(4)\cos(A).$$

Simplify both sides with arithmetic to get

$$25 = 80 - 64\cos(A).$$

Isolating $\cos(A)$ yields

$$\frac{25-80}{-64} = \cos(A).$$

Use arithmetic to simplify the numerator to get

$$\frac{55}{64} = \cos(A).$$

To find A take $\cos^{-1}$ of both sides to get

$$A = \cos^{-1}\left(\frac{55}{64}\right) \overset{\text{table}}{=} 30.8^\circ.$$

Find B
Using the law of cosines,

$$8^2 = 4^2 + 5^2 - 2(4)(5)\cos(B).$$

Isolate $\cos(B)$ to get

$$-\frac{23}{40} = \cos(B).$$

Take $\cos^{-1}$ to get

$$B = \cos^{-1}\left(-\frac{23}{40}\right) \overset{\text{table}}{=} 125.1^\circ$$

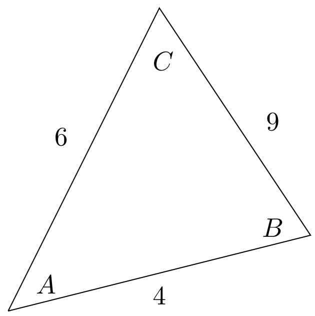
Find C
We could use the law of cosines here. But we will find C using the sum of angles equaling 180°:

$$30.8^\circ + 125.1^\circ + C = 180^\circ.$$

Solving for C yields

$$C = 180^\circ - 30.8^\circ - 125.1^\circ = 24.1^\circ.$$

6. (15 points) Find the area of the given triangle:



Solution: We will proceed using Heron's formula. Find the semiperimeter:

$$s = \frac{1}{2}(6+4+9) = \frac{19}{2}.$$

Now using Heron's formula, the area is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{19}{2}\left(\frac{19}{2}-9\right)\left(\frac{19}{2}-6\right)\left(\frac{19}{2}-4\right)}.$$

7. (10 points) (a) (5 points) Convert the polar coordinates $(r,\theta) = (-3,\pi)$ into rectangular coordinates (x,y).
   Solution: Recall polar coordinates

$$\begin{cases} x = r\cos(\theta) \\ y = r\sin(\theta) \end{cases}$$

   Plugging in the known information $r = -3$ and $\theta = \pi$ yields

$$\begin{cases} x = -3\cos(\pi) = 3 \\ y = -3\sin(\pi) = 0. \end{cases}$$

   Therefore the given polar point is $(3,0)$ in rectangular coordinates.

   (b) (5 points) Convert the rectangular coordinates $(x,y) = (\sqrt{3},1)$ into polar coordinates (r,θ).
   Solution: Using the formulas for polar coordinates

$$\begin{cases} r^2 = x^2 + y^2 \\ \theta = \tan^{-1}\left(\frac{y}{x}\right), \end{cases}$$

   we plug in the values $x = \sqrt{3}$ and $y = 1$ to get

$$\begin{cases} r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2 \\ \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}. \end{cases}$$

   Therefore the given rectangular point $(\sqrt{3},1)$ can be expressed using the polar coordinates $\left(2,\frac{\pi}{6}\right)$.

$\sin^{-1}\left(\frac{4\sin(20^\circ)}{6}\right) = 13.2^\circ$	$\sin^{-1}\left(\frac{6}{4\sin(20^\circ)}\right)$ **DNE**
$\sin^{-1}\left(\frac{6\sin(20^\circ)}{4}\right) = 30.9^\circ$	$\sin^{-1}\left(\frac{4}{6\sin(20^\circ)}\right)$ **DNE**
$\sin^{-1}\left(\frac{4}{5\sin(95^\circ)}\right) 53.4^\circ$	$\sin^{-1}\left(\frac{5\sin(95^\circ)}{4}\right)$ **DNE**
$\sin^{-1}\left(\frac{4\sin(95^\circ)}{5}\right) = 52.8^\circ$	$\sin^{-1}\left(\frac{5}{4\sin(95^\circ)}\right)$ **DNE**
$\cos^{-1}\left(\frac{73}{80}\right) = 24.2^\circ$	$\cos^{-1}\left(\frac{9}{64}\right) = 81.9^\circ$
$\sin^{-1}\left(\frac{4}{5\sin(24.2^\circ)}\right)$ **DNE**	$\sin^{-1}\left(\frac{5\sin(81.9^\circ)}{4}\right)$ **DNE**
$\sin^{-1}\left(\frac{4\sin(24.2^\circ)}{5}\right) = 19.1^\circ$	$\sin^{-1}\left(\frac{4\sin(81.9^\circ)}{5}\right) = 52.8^\circ$
$\sin^{-1}\left(\frac{5}{4\sin(24.2^\circ)}\right)$ **DNE**	$\sin^{-1}\left(\frac{5}{4\sin(81.9^\circ)}\right)$ **DNE**
$\sin^{-1}\left(\frac{5\sin(24.2^\circ)}{4}\right) = 30.8^\circ$	$\sin^{-1}\left(\frac{4}{5\sin(81.9^\circ)}\right) = 53.9^\circ$
$\sin^{-1}\left(2\sin(24.2^\circ)\right) = 55.1^\circ$	$\sin^{-1}\left(2\sin(81.9^\circ)\right)$ **DNE**
$\sin^{-1}\left(\frac{2}{\sin(24.2^\circ)}\right)$ **DNE**	$\sin^{-1}\left(\frac{\sin(81.9^\circ)}{2}\right) = 29.7^\circ$
$\sin^{-1}\left(\frac{\sin(24.2^\circ)}{2}\right) = 11.83^\circ$	$\sin^{-1}\left(\frac{2}{\sin(81.9^\circ)}\right)$ **DNE**
$\sin^{-1}\left(\frac{1}{2\sin(24.2^\circ)}\right)$ **DNE**	$\sin^{-1}\left(\frac{1}{2\sin(81.9^\circ)}\right) = 30.3^\circ$
$\cos^{-1}\left(-\frac{23}{40}\right) = 125.1^\circ$	$\cos^{-1}\left(\frac{23}{40}\right) = 54.9^\circ$
$\sin^{-1}\left(\frac{5\sin(125.1^\circ)}{8}\right) = 30.8^\circ$	$\sin^{-1}\left(\frac{8\sin(54.9^\circ)}{5}\right)$ **DNE**
$\sin^{-1}\left(\frac{5}{8\sin(125.1^\circ)}\right) = 49.8^\circ$	$\sin^{-1}\left(\frac{5}{8\sin(54.9^\circ)}\right) = 49.8^\circ$
$\sin^{-1}\left(\frac{8}{5\sin(125.1^\circ)}\right)$ **DNE**	$\sin^{-1}\left(\frac{5\sin(54.9^\circ)}{8}\right) = 30.8^\circ$
$\sin^{-1}\left(\frac{8\sin(125.1^\circ)}{5}\right)$ **DNE**	$\sin^{-1}\left(\frac{8}{5\sin(54.9^\circ)}\right)$ **DNE**
$\sin^{-1}\left(\frac{4}{8\sin(125.1^\circ)}\right) = 37.7^\circ$	$\sin^{-1}\left(\frac{4\sin(54.9^\circ)}{8}\right) = 24.1^\circ$
$\sin^{-1}\left(\frac{8\sin(125.1^\circ)}{4}\right)$ **DNE**	$\sin^{-1}\left(\frac{4}{8\sin(54.9^\circ)}\right) = 37.7^\circ$
$\sin^{-1}\left(\frac{8}{4\sin(125.1^\circ)}\right)$ **DNE**	$\sin^{-1}\left(\frac{8\sin(54.9^\circ)}{4}\right)$ **DNE**
$\sin^{-1}\left(\frac{4\sin(125.1^\circ)}{8}\right) = 24.1^\circ$	$\sin^{-1}\left(\frac{8}{4\sin(54.9^\circ)}\right)$ **DNE**

$\cos^{-1}\left(\frac{23}{400}\right) = 93.3^\circ$	$\cos^{-1}\left(\frac{55}{64}\right) = 30.8^\circ$
$\sin^{-1}\left(\frac{4}{5\sin(93.3^\circ)}\right) = 53.3^\circ$	$\sin^{-1}\left(\frac{4}{5\sin(30.8^\circ)}\right)$ **DNE**
$\sin^{-1}\left(\frac{4\sin(93.3^\circ)}{5}\right) = 53.0^\circ$	$\sin^{-1}\left(\frac{4\sin(30.8^\circ)}{5}\right) = 24.2^\circ$
$\sin^{-1}\left(\frac{5}{4\sin(93.3^\circ)}\right)$ **DNE**	$\sin^{-1}\left(\frac{5}{4\sin(30.8^\circ)}\right)$ **DNE**
$\sin^{-1}\left(\frac{5\sin(93.3^\circ)}{4}\right)$ **DNE**	$\sin^{-1}\left(\frac{5\sin(30.8^\circ)}{4}\right) = 39.8^\circ$
$\sin^{-1}\left(\frac{8}{5\sin(93.3^\circ)}\right)$ **DNE**	$\sin^{-1}\left(\frac{8}{5\sin(30.8^\circ)}\right)$ **DNE**
$\sin^{-1}\left(\frac{8\sin(93.3^\circ)}{5}\right)$ **DNE**	$\sin^{-1}\left(\frac{8\sin(30.8^\circ)}{5}\right) = 55.0^\circ$
$\sin^{-1}\left(\frac{5}{8\sin(93.3^\circ)}\right) = 38.8^\circ$	$\sin^{-1}\left(\frac{5}{8\sin(30.8^\circ)}\right)$ **DNE**
$\sin^{-1}\left(\frac{5\sin(93.3^\circ)}{8}\right) = 38.6^\circ$	$\sin^{-1}\left(\frac{5\sin(30.8^\circ)}{8}\right) = 18.7^\circ$