MATH 1112 - EXAM 4 FALL 2016
SOLUTION

Friday 18 November 2016
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (10 points) Determine if the function is one-to-one. If it is one-to-one, find its inverse function.
   (a) (5 points) $f(x) = (x+2)^2$
   Solution: This function is not one-to-one (it does not pass the horizontal line test).
   (b) (5 points) $f(x) = 3x + 5$
   Solution: This function is one-to-one. To find its inverse consider

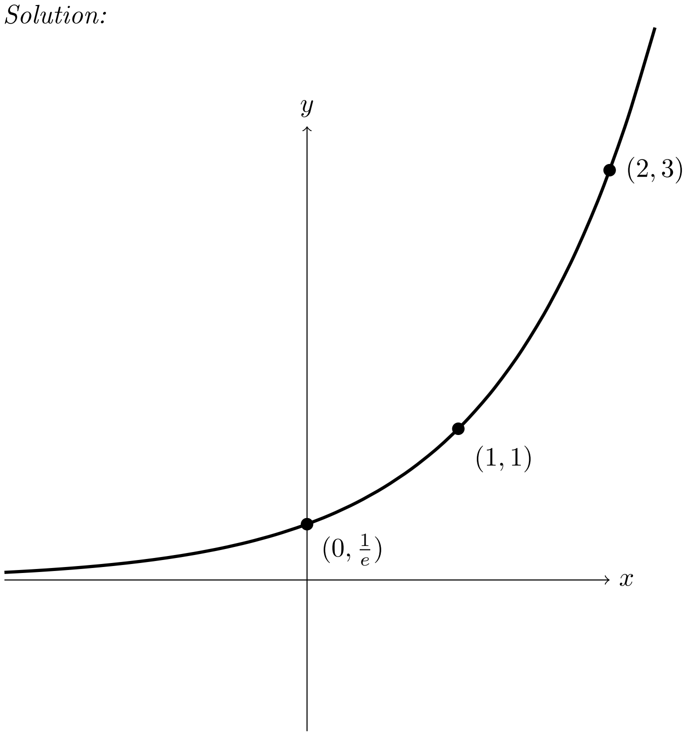
$$y = 3x + 5$$

   and solve for x by subtracting 5 and dividing by 3 to get

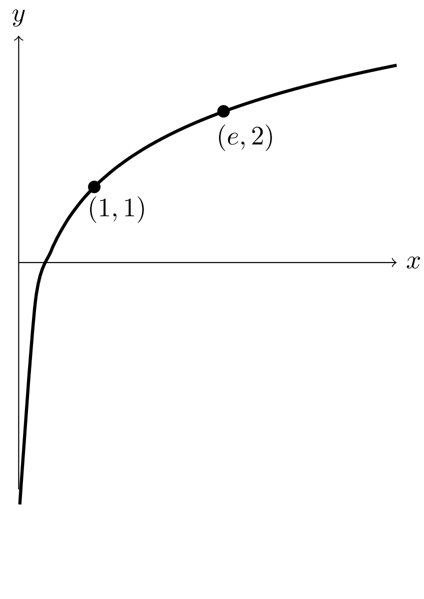
$$x = \frac{y-5}{3}.$$

   This shows that $f^{-1}(y) = \dfrac{y-5}{3}$.

2. (10 points) Sketch the graph of the given function.
   (a) (5 points) $f(x) = e^{x-1}$
   Solution:



   (b) (5 points) $f(x) = \ln(x) + 1$



3. (15 points) Find an exact value for the logarithm.

   (a) (5 points) $\log_4(16)$
   Solution: Calculate
   $$\log_4(16) = \log_4(4^2) = 2.$$

   (b) (5 points) $\log_3(27)$
   Solution: Calculate
   $$\log_3(27) = \log_3(3^3) = 3.$$

   (c) (5 points) $\ln(\sqrt{e})$
   Solution: Calculate
   $$\ln(\sqrt{e}) = \ln(e^{\frac{1}{2}}) = \frac{1}{2}.$$

4. (15 points) Write as a sum or difference of logarithms.

   (a) (7 points) $\log_a(10x^2y^3)$
   Solution: Calculate
   $$\begin{aligned}\log_a(10x^2y^3) &= \log_a(10) + \log_a(x^2) + \log_a(y^3)\\ &= \log_a(10) + 2\log_a(x) + 3\log_a(y).\end{aligned}$$

   (b) (8 points) $\log_a\left(\sqrt{\frac{a^4b^3}{3}}\right)$
   Solution: Calculate
   $$\begin{aligned}\log_a\left(\sqrt{\frac{a^4b^3}{3}}\right) &= \log_a\left(\left(\frac{a^4b^3}{3}\right)^{\frac{1}{2}}\right)\\ &= \frac{1}{2}\log_a\left(\frac{a^4b^3}{3}\right)\\ &= \frac{1}{2}\left[\log_a(a^4) + \log_a(b^3) - \log_a(3)\right]\\ &= \frac{1}{2}\left[4 + 3\log_a(b) - \log_a(3)\right]\end{aligned}$$

5. (15 points) Express as a single logarithm.

   (a) (7 points) $2\log_a(x) + \log_a(y)$
   Solution: Calculate
   $$2\log_a(x) + \log_a(y) = \log_a(x^2) + \log_a(y) = \log_a(x^2y).$$

   (b) (8 points) $\frac{1}{2}\log_a(x) + 4\log_a(y) - 3\log_a(x)$
   Solution: Calculate
   $$\begin{aligned}\frac{1}{2}\log_a(x) + 4\log_a(y) - 3\log_a(x) &= \log_a(\sqrt{x}) + \log_a(y^4) - \log_a(x^3)\\ &= \log_a\left(\frac{\sqrt{x}y^4}{x^3}\right).\end{aligned}$$

6. (10 points) Under certain conditions an initial population of 1000 bacteria grows exponentially with a growth rate of 5% per hour.

   (a) (5 points) Find the exponential growth function.
   Solution: Note that $5\% = \frac{5}{100} = 0.05$. The exponential growth function is
   $$P(t) = 1000e^{0.05t}.$$

(b) (5 points) Find the doubling time.
Solution: The doubling time is the solution t to the equation

$$2(1000) = 1000e^{0.05t}.$$

Dividing by 1000 yields

$$2 = e^{0.05t}.$$

Now take ln of both sides to get

$$\ln(2) = 0.05t.$$

Finally, divide by 0.05 to get

$$t = \frac{\ln(2)}{0.05}.$$

7. (25 points) Solve the equation.

(a) (6 points) $13e^t = 11$
Solution: First divide by 13 to get

$$e^t = \frac{11}{13}.$$

Now take ln of both sides to get

$$\ln(e^t) = \ln\left(\frac{11}{13}\right).$$

On the left the ln and the base e cancel to yield

$$t = \ln\left(\frac{11}{13}\right).$$

(b) (6 points) $\log_2(x+1) = 3$
Solution: Plug both sides into 2^x to get

$$2^{\log_2(x+1)} = 2^3.$$

On the left, the 2 and the $\log_2$ cancel and the right side reduces to 8 to yield

$$x + 1 = 8.$$

Therefore $x = 7$.

(c) (6 points) $\ln(x) - \ln(x-1) = \ln(2)$
Solution: Use the fact that the sum of logs is the log of the product to combine the logarithms on the left, yielding

$$\ln(x(x-1)) = \ln(2).$$

Now plug both sides into e^x to get

$$e^{\ln(x(x-1))} = e^{\ln(2)}.$$

The e and ln cancel each other on both sides yielding

$$x(x-1) = 2.$$

This is the equation

$$x^2 - x = 2,$$

or

$$x^2 - x - 2 = 0.$$

Solve this quadratic any way you wish (it factors or you could use quadratic formula) to get solutions $x = -1$ and $x = 2$. (Note that $x = -1$ cannot be plugged into $\ln(x)$, so we should drop that solution).

(d) (7 points) $2^{x+3} = 5^{x+1}$
Solution: This equation can be solved by taking *any* logarithm. We will provide a solution taking $\log_2$ of both sides. Do this to get

$$\log_2(2^{x+3}) = \log_2(5^{x+1}).$$

On the left, the 2 and $\log_2$ cancel and on the right pull the power in the front yielding

$$x + 3 = (x + 1)\log_2(5).$$

Now collect all x's on the left and all non-x's on the right:

$$x - \log_2(5)x = \log_2(5) - 3.$$

Factor the x on the left side to get

$$x\,(1 - \log_2(5)) = \log_2(5) - 3.$$

Now divide both sides by $1 - \log_2(5)$ to get

$$x = \frac{\log_2(5) - 3}{1 - \log_2(5)}.$$

Note: if you chose a different logarithm, your answer may look different than this!