

MATH 1112 - EXAM 3 - FALL 2016

SOLUTION

28 October 2016

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Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (21 points) Write in the form $a + bi$, where a and b are real numbers.

(a) (3 points) $\sqrt{-41}$

Solution: $\sqrt{41}i$

(b) (6 points) $(2 + 3i) - (1 - 4i)$

Solution: Calculate

$$(2 + 3i) - (1 - 4i) = (2 - 1) + (3 - (-4))i = 1 + 7i.$$

(c) (6 points) $(3 - i)(4 + 2i)$

Solution: Calculate

$$\begin{aligned}(3 - i)(4 + 2i) &= 12 + 6i - 4i - 2i^2 \\ &= 12 + 2i + 2 \\ &= 14 + 2i.\end{aligned}$$

(d) (6 points) $\frac{2 + 3i}{1 - i}$

Solution: Calculate

$$\begin{aligned}\frac{2 + 3i}{1 - i} &= \left(\frac{2 + 3i}{1 - i}\right) \left(\frac{1 + i}{1 + i}\right) \\ &= \frac{(2 + 3i)(1 + i)}{(1 - i)(1 + i)} \\ &= \frac{2 + 2i + 3i + 3i^2}{2 + 2i + 3i + 3i^2} \\ &= \frac{1 + i - i - i^2}{2 + 5i - 3} \\ &= \frac{1 - (-1)}{-1 + 5i} \\ &= \frac{2}{-1 + 5i} \\ &= -\frac{1}{2} + \frac{5}{2}i.\end{aligned}$$

2. (15 points) Simplify.

(a) (5 points) i^5

Solution: Compute

$$i^5 = ii^4 = i(i^2)^2 = i(-1)^2 = i.$$

(b) (5 points) i^{53}

Solution: Compute

$$i^{53} = ii^{52} = i(i^2)^{26} = i(-1)^{26} = i.$$

(c) (5 points) i^{103}

Solution: Compute

$$i^{103} = ii^{102} = i(i^2)^{51} = i(-1)^{51} = -i.$$

3. (22 points) Solve the following equations in any way you wish.

(a) (5 points) $(x + 3)(x - 1) = 0$

Solution: Using the zero-product property of algebra, we get $x = -3$ and $x = 1$.

(b) (5 points) $x^2 + 16 = 0$

Solution: Subtract 16 to get

$$x^2 = -16.$$

Now take square roots to get

$$x = \pm\sqrt{-16} = \pm 4i.$$

(c) (6 points) $x^2 + 5x + 1 = 0$

Solution: Use the quadratic formula with $a = 1$, $b = 5$, and $c = 1$ to see

$$x = \frac{-5 \pm \sqrt{25 - 4(1)(1)}}{2} = -\frac{5}{2} \pm \frac{\sqrt{21}}{2}.$$

(d) (6 points) $3x^2 - x + 5 = 0$

Solution: Use the quadratic formula with $a = 3$, $b = -1$, and $c = 5$ to get

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(5)}}{2(3)} = \frac{1 \pm \sqrt{-59}}{6} = \frac{1}{6} \pm \frac{\sqrt{59}}{6}i.$$

4. (21 points) Analyze the following function:

$$f(x) = -3(x - 4)^2 - 5$$

(a) (4 points) What is the vertex of f ?

Solution: The vertex is $(4, -5)$.

(b) (3 points) What is the axis of symmetry?

Solution: The axis of symmetry is $x = 4$.

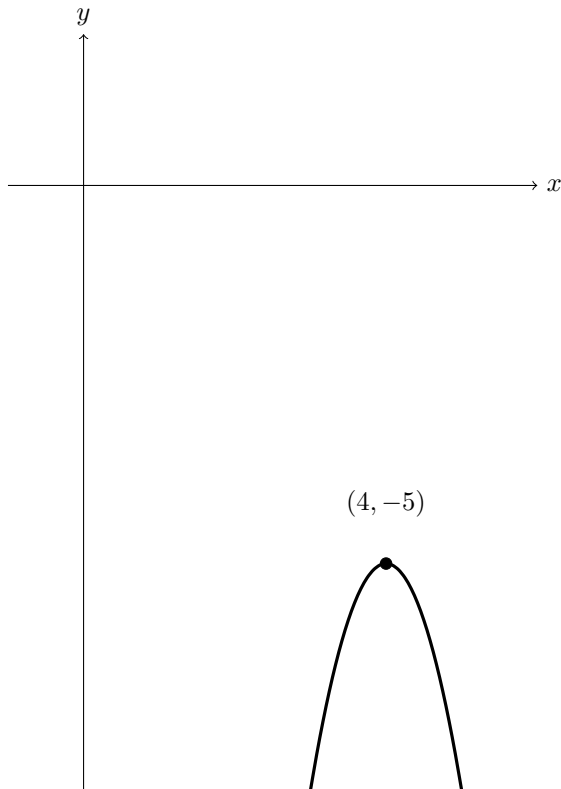
(c) (4 points) What is the minimum (if it exists)?

Solution: There is no minimum.

(d) (4 points) What is the maximum (if it exists)?

Solution: The maximum occurs at $x = 4$ and it has value -5 .

(e) (6 points) Graph $y = f(x)$ based on the information found in parts (a)–(d). *Solution:*



5. (21 points) Analyze the following function:

$$f(x) = 2x^2 - 3x + 1$$

(a) (4 points) What is the vertex of f ?

Solution: Completing the square on f yields

$$f(x) = 2\left(x - \frac{3}{4}\right)^2 - \frac{1}{8}.$$

From this it is clear that the vertex is $\left(\frac{3}{4}, -\frac{1}{8}\right)$.

(b) (3 points) What is the axis of symmetry?

Solution: The axis of symmetry is $x = \frac{3}{4}$.

(c) (4 points) What is the minimum (if it exists)?

Solution: The minimum is $-\frac{1}{8}$ and it occurs as $x = \frac{3}{4}$.

(d) (4 points) What is the maximum (if it exists)?

Solution: There is no maximum.

(e) (6 points) Graph $y = f(x)$ based on the information found in parts (a)–(d).

