MATH 1112 - EXAM 1 - FALL 2016
SOLUTION

Friday 9 September 2016
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (15 points) For the points $P = (-1, 2)$ and $Q = (4, 6)$:

   (a) (3 points) Find the distance between P and Q.
   Solution: The distance formula (pg. 8 in the book) for the distance between a point $P = (x_1, y_1)$ and a point $Q = (x_2, y_2)$ is given by

   $$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

   For our particular P and Q this yields

   $$d(P, Q) = \sqrt{(-1 - 4)^2 + (2 - 6)^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25 + 16} = \sqrt{41}.$$

   (b) (3 points) Find the midpoint between P and Q.
   Solution: The midpoint (pg. 10) between the point $P = (x_1, y_1)$ and the point $Q = (x_2, y_2)$ is the point

   $$\text{midpoint}(P, Q) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

   For our particular P and Q, this yields

   $$\text{midpoint}(P, Q) = \left(\frac{-1 + 4}{2}, \frac{2 + 6}{2}\right) = \left(\frac{3}{2}, \frac{8}{2}\right) = \left(\frac{3}{2}, 4\right).$$

   (c) (3 points) Find an equation of the line containing the points P and Q.
   Solution: We find the slope of this line by

   $$m = \frac{6 - 2}{4 - (-1)} = \frac{4}{5}.$$

   Now we use the point-slope form of the equation of the line (pg. 51) with the point $Q = (4, 6)$ (it's ok to use P here as well) to get

   $$y - 6 = \frac{4}{5}(x - 4).$$

   If you had used P you would have gotten

   $$y - 2 = \frac{4}{5}(x - (-1)),$$

   which is an equivalent equation. Indeed, both of these are also equivalent to the "slope-intercept" form

   $$y = \frac{4}{5}x + \frac{14}{5},$$

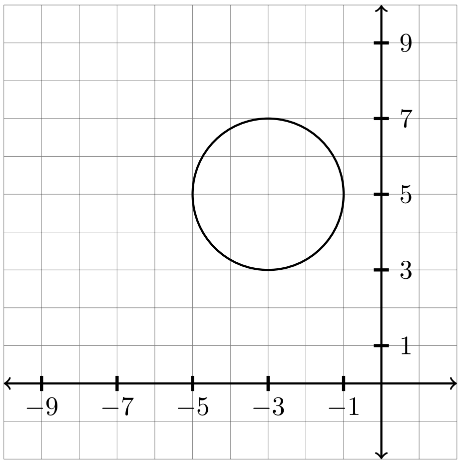
   which is a third equivalent way to write the equation.

   (d) (6 points) Give the slope and the y-intercept of the line you found in part (c).
   Solution: We saw above that the slope-intercept form of the equation of the line (pg. 41) in part (c) is

   $$y = \frac{4}{5}x + \frac{14}{5}.$$

   In other words, the slope is $\frac{4}{5}$ and the y-intercept is $\left(0, \frac{14}{5}\right)$.

2. (10 points) Write an equation for the circle with center $(-5, 3)$ and radius 2 **and** draw this circle in the provided grid.



The equation of a circle with center (h, k) and radius r is (pg. 11)

$$(x - h)^2 + (y - k)^2 = r^2.$$

We are told $(h, k) = (-5, 3)$ and $r = 2$, so our circle has equation

$$(x - (-5))^2 + (y - 3)^2 = 2^2,$$

or, simplified,

$$(x + 5)^2 + (y - 3)^2 = 4.$$

3. (10 points) Find an equation of the following lines:

(a) (5 points) the line passing through the point $(3, -2)$ and parallel to $3x + 4y = 5$.
Solution: Such a line must have the same slope as $3x + 4y = 5$ and also pass through $(3, -2)$. The fastest way (but not only) to find the slope of $3x + 4y = 5$ is to write it in slope-intercept form, i.e. solve for y. Do this by first subtracting $3x$ to get

$$4y = -3x + 5,$$

and then dividing by 4 to get

$$y = -\frac{3}{4}x + \frac{5}{4}.$$

From this formula, we can read off the slope of the line $3x + 4y = 5$ as $m = -\frac{3}{4}$. We want to find an equation for the line with slope $m = -\frac{3}{4}$ passing through the point $(3, -2)$. We use the point-slope form of the line to get

$$y - (-2) = -\frac{3}{4}(x - 3),$$

or

$$y + 2 = -\frac{3}{4}(x - 3).$$

Which we could also write in the slope intercept form by subtracting 2 and distributing $-\frac{3}{4}$ to get

$$y = -\frac{3}{4}x + \frac{9}{4} - 2 = -\frac{3}{4}x + \frac{9}{4} - \frac{8}{4} = -\frac{3}{4}x + \frac{1}{4}.$$

(b) (5 points) the line passing through the point $(3, -2)$ and perpendicular to $3x + 4y = 5$.
Solution: The line perpendicular to $3x + 4y = 5$ has negative reciprocal slope of it. Since the slope of $3x + 4y = 5$ is $-\frac{3}{4}$ as discussed above, the line perpendicular to $3x + 4y = 5$ must have slope $m = \frac{4}{3}$. Using the point-slope form of the equation of a line, the line with slope $\frac{4}{3}$ passing through $(3, -2)$ is

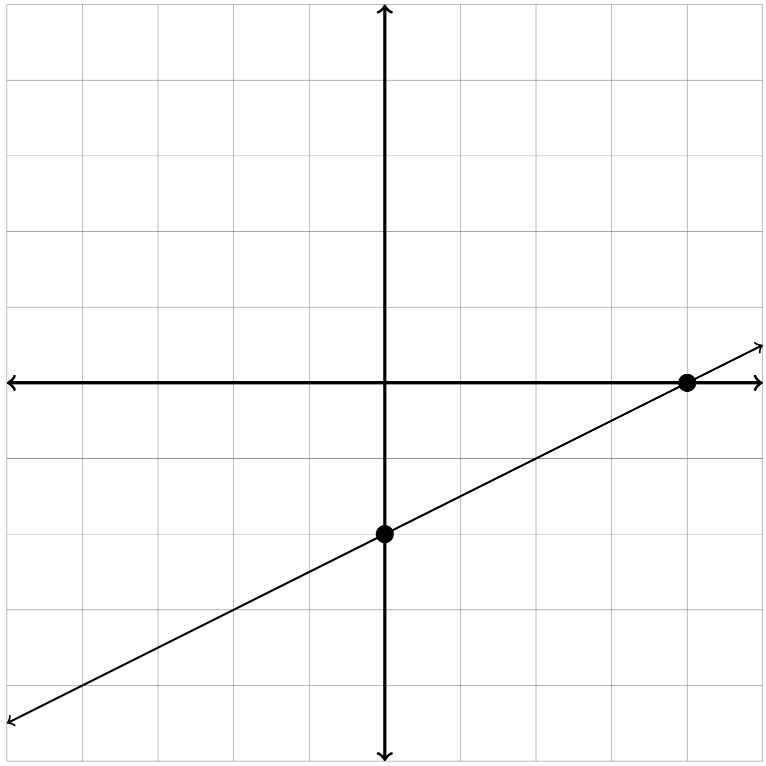
$$y - (-2) = \frac{4}{3}(x - 3),$$

or

$$y + 2 = \frac{4}{3}(x - 3).$$

4. (10 points) Find all intercepts and then graph the equation. If an intercept does not exist, state so:

$$2x - 4y = 8.$$



Solution: To find the x-intercept, set $y = 0$ in the equation to get

$$2x = 8,$$

and solve it to get $x = 4$. Therefore the x-intercept is $(4, 0)$. To find the y-intercept, set $x = 0$ in the equation to get

$$-4y = 8,$$

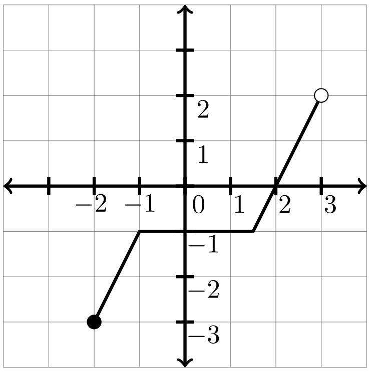
and solve it to get $y = -2$. Therefore the y-intercept is $(0, -2)$. To plot, we place the intercepts on the graph and then draw the line connecting them.

x-intercept(s): $(4, 0)$ y-intercept(s): $(0, -2)$

5. (10 points) What is the domain of the function $g(x) = \dfrac{5x}{(x+1)(x-5)}$?

Solution: The domain of g is all real numbers except where the denominator is zero. The denominator is zero whenever $x + 1 = 0$, i.e. $x = -1$ or $x - 5 = 0$, i.e. $x = 5$. Therefore the domain of g is "all real numbers except for -1 or 5" or an equivalent expression.

6. (10 points) State the domain and range of the function with the following graph.



Domain: $[-2, 3)$

Range: $[-3, 2)$

7. (15 points) Solve the following **and** graph the solution.

(a) (5 points) $2x + 1 < 3$

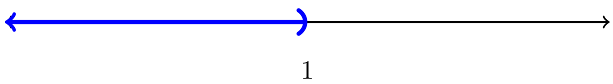
Solution: Solve this inequality by first subtracting 1 to get

$$2x < 2,$$

and then dividing by 2 to get

$$x < 1.$$

The graph is



(b) (5 points) $-2x - 2 \geq 6$

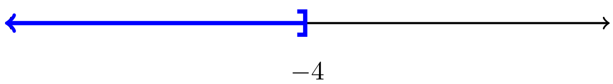
Solution: First add 2 to get

$$-2x \geq 8.$$

Divide both sides by -2 (recall to flip the inequality) to get

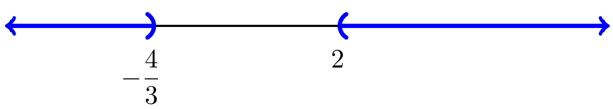
$$x \leq -4.$$

The graph is



(c) (5 points) $3x - 1 < -5$ or $3x - 1 > 5$

Solution: Solve the first inequality to get $x < -\frac{4}{3}$ and solve the second to get $x > \frac{6}{3} = 2$. To graph the inequality we plot both of these on the same line:



8. (10 points) Recall that the interest I on a principal P invested at an interest rate r for t years is $I = Prt$. How much principal would you have to invest at a 10% interest rate for 2 years if you want to make \$100 in interest?

Solution: We are told $I = \$50$, $r = 10\% = \frac{10}{100} = \frac{1}{10}$, and $t = 2$ years. Plugging this information into the equation $I = Prt$ yields

$$100 = P\frac{2}{10},$$

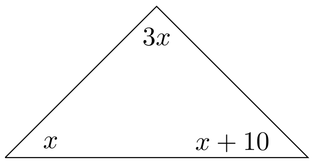
or

$$100 = \frac{P}{5}.$$

Solving this for P yields

$$P = \$500.$$

9. (10 points) In a triangle, the top angle is three times as large as the left angle which has measure x. The right angle is 10° more than angle x. Find the measures of the three angles. (Hint: the sum of the angles in a triangle add to 180°.)



Solution: The sum of the angles of the triangle are 180 and so

$$3x + x + (x + 10) = 180.$$

Simplifying the left hand side yields

$$5x + 10 = 180.$$

Subtract 10 to get

$$5x = 170.$$

Now divide by 5 to get

$$x = \frac{170}{5} = 34.$$

The other two angles are $34 + 10 = 44$ and $3 \cdot 34 = 102$.