MATH 1011 - EXAM 5 FALL 2016 SOLUTION

Thursday 17 November 2016
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Instructions:

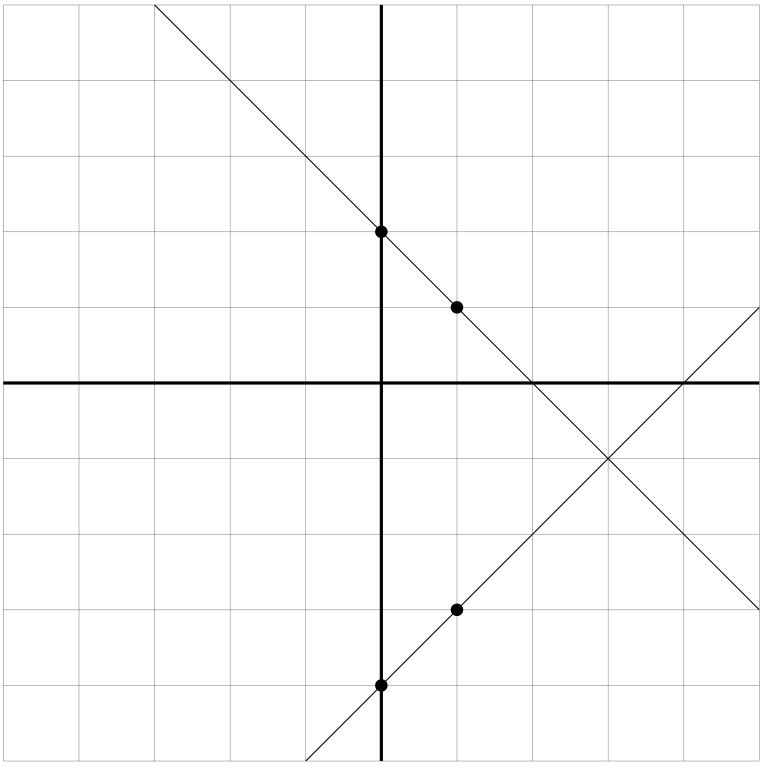
- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (8 points) Solve the following system of equations by graphing (for each equation, find two solutions and connect the dots to get the correct graph):

$$\begin{cases} x + y &= 2 \quad (i) \\ -x + y &= -4 \quad (ii). \end{cases}$$

Solution: To plot (i) let us first find two solutions. To find one, let $x = 0$ giving us $y = 2$ and hence we will plot $(0, 2)$. To find the other, let $x = 1$ giving us $1 + y = 2$ and so $y = 1$ and hence we will plot $(1, 1)$ and connect it to $(0, 2)$.

To plot (ii) let $x = 0$ yielding $y = -4$ so we plot $(0, -4)$. Now let $x = 1$ yielding $-1 + y = -4$ and $y = -3$, so we will plot $(1, -3)$ and connect it to $(0, -4)$.



From the graph we see that the point of intersection (i.e. the solution) is at $(3, -1)$.

2. (14 points) VIP tickets for a concert cost \$20 and regular tickets cost \$10. A total of 110 tickets were sold earning \$1200. How many of each type were sold?
Solution: Let x denote the number of VIP tickets and let y denote the number of regular tickets. To say 110 tickets were sold means $x + y = 110$. To say that \$1200 was earned means that $20x + 10y = 1200$. Therefore we must solve the system of equations

$$\begin{cases} x + y = 110 \quad (i) \\ 20x + 10y = 1200 \quad (ii). \end{cases}$$

Solve (i) for y to get $y = 110 - x$. Plug this into (ii) to get

$$20x + 10(110 - x) = 1200.$$

Distribute the 10 to get

$$20x + 1100 - 10x = 1200.$$

Subtract 1100 and combine like terms on the left to get

$$10x = 100,$$

therefore

$$x = 10.$$

Plug this value of x into our equation for y to get

$$y = 110 - 10 = 100.$$

3. (14 points) Solve the following systems of equations any way you wish.

   (a) (7 points) $\begin{cases} 3x - 2y = -7 & (i) \\ x + y = 1 & (ii) \end{cases}$

   Solution: Solve (ii) for x to get

   $$x = 1 - y.$$

   Plug this into (i) to get

   $$3(1 - y) - 2y = -7.$$

   Distribute the 3 to get

   $$3 - 3y - 2y = -7.$$

   Subtract 3 and combine like terms on the left to get

   $$-5y = -10.$$

   Therefore

   $$y = 2.$$

   Plug this value of y into our equation for x to get

   $$x = 1 - 2 = -1.$$

   (b) (7 points) $\begin{cases} 2x + y = 1 & (i) \\ 4x + 2y = 2 & (ii) \end{cases}$

   Solution: Solve (i) for y to get

   $$y = 1 - 2x.$$

   Plug this into (ii) to get

   $$4x + 2(1 - 2x) = 2.$$

   Distribute the 2 on the left to get

   $$4x + 2 - 4x = 2.$$

   Simplify the left to get

   $$2 = 2.$$

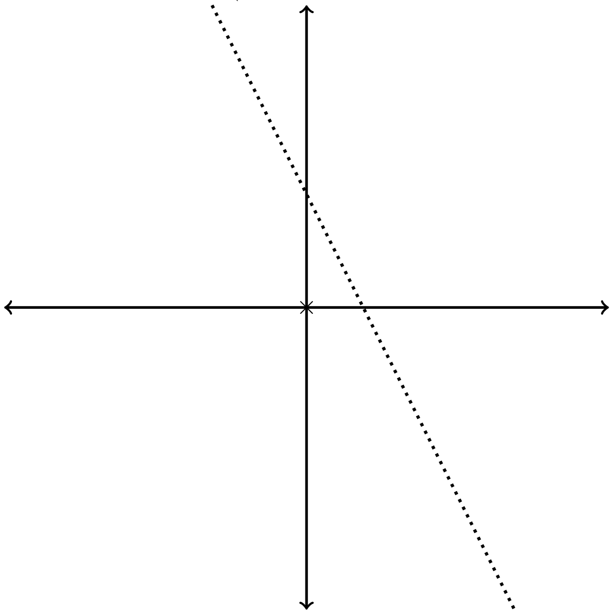
   This equation is **always true**, and so this system of equations has infinitely many solutions.

4. (14 points) Graph the given inequalities.

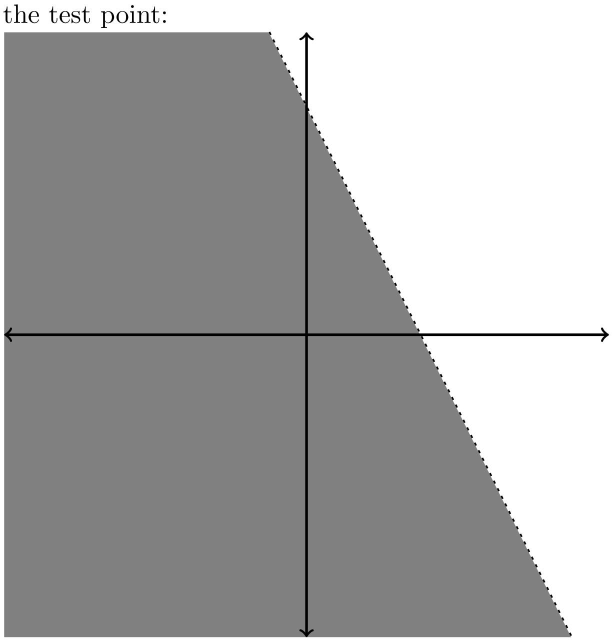
(a) (7 points) $2x + y > 3$

Solution:

First draw the line (as a dotted line because we have $>$ not $\geq$):

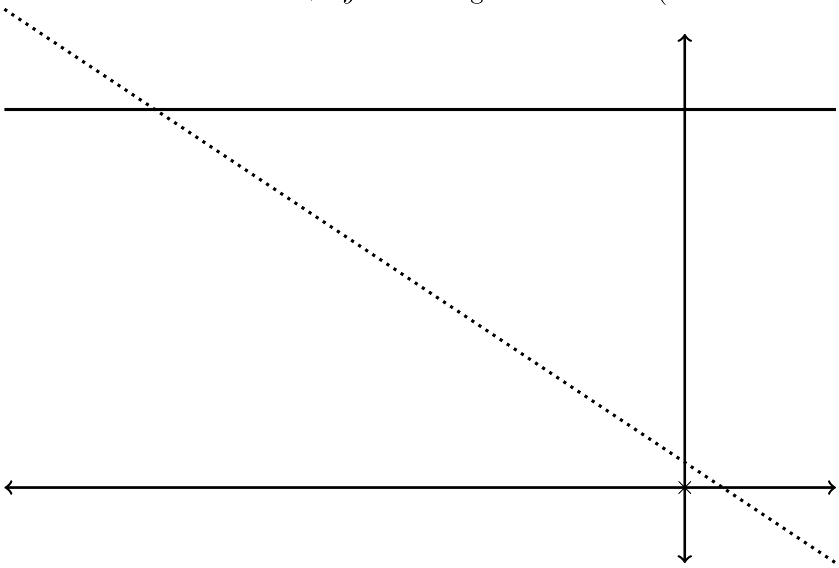


We use the test point $(0, 0)$ to decide how to shade. Plug $x = 0$ and $y = 0$ into the inequality and observe that $2(0) + 0 > 3$ is **false**. Therefore we shade the half of the plane not containing the test point:

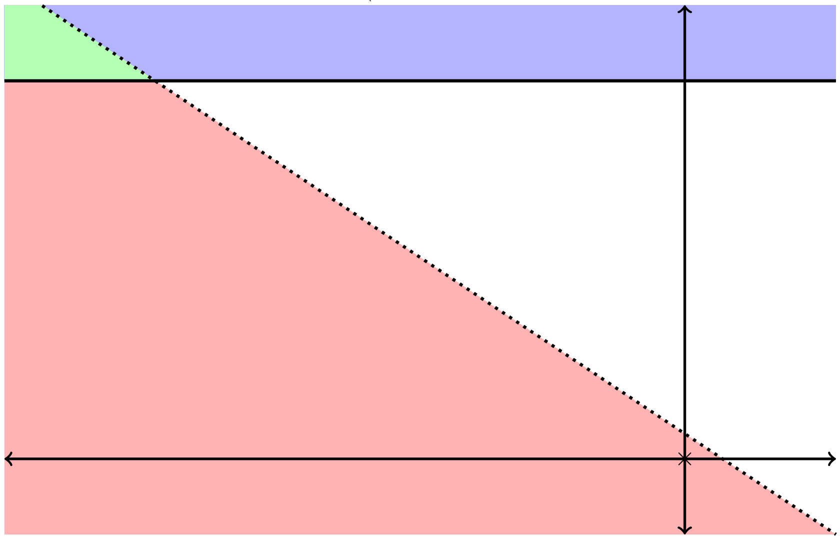


(b) (7 points) $\begin{cases} y \geq 5 \\ 2x + 3y < 1 \end{cases}$

Solution: We graph the line $y = 5$ with a solid line (because that one has a $\geq$ not a $>$) and we draw the line $2x + 3y = 1$ using a dotted line (because that one has a $<$, not a $\leq$):



We use the test point $(0, 0)$ and plug it into each inequality. Plugging it into $y \geq 5$ yields $0 \geq 5$ which is **false**. Therefore I will shade the half of the line $y = 5$ not containing the test point in blue. Similarly, I plug the test point into the inequality $2x + 3y < 1$ and I end up with $0 + 0 < 1$ which is **true**. Therefore I will shade the half of the line $2x + 3y = 1$ containing the test point red. The common intersection (i.e. solution of the system of inequalities) is shown in green.



5. (20 points) Simplify the radical expression

   (a) (5 points) $\sqrt{48}$
   Solution: Calculate
$$\sqrt{48} = \sqrt{4 \cdot 12} = 2\sqrt{12} = 2\sqrt{4 \cdot 3} = 4\sqrt{3}.$$

   (b) (5 points) $\sqrt{36x^4}$
   Solution: Calculate
$$\sqrt{36x^4} = \sqrt{36}\sqrt{x^4} = 6x^2.$$

   (c) (5 points) $\sqrt{\dfrac{18x^2}{100x^3}}$
   Solution: Calculate
$$\begin{aligned}\sqrt{\frac{18x^2}{100x^3}} &= \frac{\sqrt{18x^2}}{\sqrt{100x^3}} \\ &= \frac{\sqrt{9 \cdot 2x^2}}{\sqrt{100x^2x}} \\ &= \frac{3\sqrt{2}x}{10x\sqrt{x}} \\ &= \frac{3\sqrt{2}}{10\sqrt{x}}.\end{aligned}$$

   (d) (5 points) $\sqrt{x^{21}}$
   Solution: Calculate
$$\sqrt{x^{21}} = \sqrt{xx^{20}} = \sqrt{x}\sqrt{(x^10)^2} = x^{10}\sqrt{x}.$$

6. (14 points) Rationalize the denominator.

   (a) (7 points) $\dfrac{1}{\sqrt{11}}$
   Solution: Calculate
$$\frac{1}{\sqrt{11}} = \left(\frac{1}{\sqrt{11}}\right)\left(\frac{\sqrt{11}}{\sqrt{11}}\right) = \frac{\sqrt{11}}{11}.$$

   (b) (7 points) $\dfrac{3}{5-\sqrt{2}}$
   Solution: Calculate
$$\frac{3}{5-\sqrt{2}} = \left(\frac{3}{5-\sqrt{2}}\right)\left(\frac{5+\sqrt{2}}{5+\sqrt{2}}\right) = \frac{15+3\sqrt{2}}{25-2} = \frac{15+3\sqrt{2}}{23}.$$

7. (16 points) Solve the given radical equation.

   (a) (8 points) $\sqrt{x+2} = x+2$
   Solution: Square both sides to get
$$x + 2 = (x+2)^2.$$
   Expand the right-hand side to get
$$x + 2 = x^2 + 4x + 4.$$
   Move everything to the right side to get
$$0 = x^2 + 3x + 2.$$
   Factor to get
$$0 = (x+2)(x+1),$$
   and we see the solution is $x = -2, -1$. Notice that plugging these solutions into the original equation yields true statements, and so these are really solutions.

(b) (8 points) $\sqrt{x+3} = -2$
Solution: Square both sides to get

$$x + 3 = 4.$$

Subtract 3 to get

$$x = 1.$$

Check this answer by plugging it into the original equation to get

$$\sqrt{1+3} = -2,$$

or

$$\sqrt{4} = -2,$$

but this is **false** – therefore this equation has **no solution**.