

MATH 1011 - EXAM 4 - FALL 2016

SOLUTION

Thursday 27 October 2016

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Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (15 points) For which values of x are the following expressions defined?

(a) (5 points) $\frac{2x^2 - 9x + 3}{x + 1}$

Solution: The expression is valid when

$$x + 1 \neq 0,$$

or

$$x \neq -1.$$

(b) (5 points) $\frac{x^2 + 7}{(x + 3)(x - 6)}$

Solution: The expression is valid when

$$(x + 3)(x - 6) \neq 0,$$

or

$$x \neq -3, 6.$$

(c) (5 points) $\frac{6x^3 + 9x + 2}{x^2 + 9x + 14}$

Solution: The expression is valid when

$$x^2 + 9x + 14 \neq 0,$$

or

$$(x + 7)(x + 2) \neq 0,$$

or

$$x \neq -7, -2.$$

2. (15 points) Find the least common multiple of . . .

(a) (5 points) 2, 6, and 7

Solution: Since $6 = 3 \cdot 2$ we see that

$$\text{lcm}(2, 6, 7) = \text{lcm}(2, 3 \cdot 2, 7) = 2 \cdot 3 \cdot 7 = 42.$$

(b) (5 points) $(x + 2)(x + 6)^2$, $(x + 2)^2(x + 6)$, and $(x + 2)(x + 6)$

Solution:

$$\text{lcm}((x + 2)(x + 6)^2, (x + 2)^2(x + 6), (x + 2)(x + 6)) = (x + 2)^2(x + 6)^2$$

(c) (5 points) $x^2 + 5x + 6$, $x^2 + 3x + 2$, and $x^2 + 7x + 12$

Solution: Since

$$x^2 + 5x + 6 = (x + 3)(x + 2),$$

$$x^2 + 3x + 2 = (x + 1)(x + 2),$$

and

$$x^2 + 7x + 12 = (x + 3)(x + 4),$$

we see that

$$\begin{aligned} \text{lcm}(x^2 + 5x + 6, x^2 + 3x + 2, x^2 + 7x + 12) \\ &= \text{lcm}((x + 3)(x + 2), (x + 2)(x + 1), (x + 4)(x + 3)) \\ &= (x + 4)(x + 3)(x + 2)(x + 1) \end{aligned}$$

3. (20 points) Add or subtract and simplify

(a) (5 points) $\frac{1}{3} + \frac{1}{7}$

Solution: The common denominator is 21, so

$$\frac{1}{3} + \frac{1}{7} = \frac{7}{21} + \frac{3}{21} = \frac{10}{21}.$$

(b) (5 points) $\frac{1}{x-1} + \frac{1}{x+3}$

Solution: The common denominator is $(x-1)(x+3)$, so

$$\frac{1}{x-1} + \frac{1}{x+3} = \frac{x+3}{(x-1)(x+3)} + \frac{x-1}{(x-1)(x+3)} = \frac{2x+2}{(x-1)(x+3)}.$$

(c) (5 points) $\frac{5}{(x+2)(x+3)} - \frac{x}{(x+1)(x+2)}$

Solution: The common denominator is $(x+1)(x+2)(x+3)$, so

$$\begin{aligned} \frac{5}{(x+2)(x+3)} - \frac{x}{(x+1)(x+2)} &= \frac{5(x+1)}{(x+1)(x+2)(x+3)} - \frac{x(x+3)}{(x+1)(x+2)(x+3)} \\ &= \frac{5x+5-x^2-3x}{(x+1)(x+2)(x+3)} \\ &= \frac{-x^2+2x+5}{(x+1)(x+2)(x+3)}. \end{aligned}$$

(d) (5 points) $\frac{1}{x^2+2x+1} + \frac{1}{x^2+6x+5}$

Solution: Since

$$x^2+2x+1 = (x+1)(x+1)$$

and

$$x^2+6x+5 = (x+3)(x+2),$$

we see that the common denominator is $(x+1)^2(x+2)(x+3)$. So,

$$\begin{aligned} \frac{1}{x^2+2x+1} + \frac{1}{x^2+6x+5} &= \frac{1}{(x+1)(x+1)} + \frac{1}{(x+3)(x+2)} \\ &= \frac{(x+3)(x+2)}{(x+1)^2(x+2)(x+3)} + \frac{(x+1)^2}{(x+1)^2(x+2)(x+3)} \\ &= \frac{x^2+5x+6}{(x+1)^2(x+2)(x+3)} + \frac{x^2+2x+1}{(x+1)^2(x+2)(x+3)} \\ &= \frac{2x^2+7x+7}{(x+1)^2(x+2)(x+3)}. \end{aligned}$$

4. (15 points) Simplify.

(a) (7 points) $\frac{\frac{1}{x}+2}{\frac{1}{x}+3}$

Solution: Get a common denominator on top and bottom then simplify the fraction of fractions:

$$\begin{aligned} \frac{\frac{1}{x}+2}{\frac{1}{x}+3} &= \frac{\frac{1+2x}{x}}{\frac{1+3x}{x}} \\ &= \left(\frac{1+2x}{x} \right) \left(\frac{x}{1+3x} \right) \\ &= \frac{1+2x}{1+3x}. \end{aligned}$$

(b) (8 points) $\frac{x-3+\frac{1}{x}}{\frac{1}{x+8}+x}$

Solution: Get a common denominator on top and bottom and then simplify the fraction of fractions:

$$\begin{aligned} \frac{x-3+\frac{1}{x}}{\frac{1}{x+8}+x} &= \frac{\frac{x^2-3x+1}{x}}{\frac{1+x^2+8x}{x+8}} \\ &= \left(\frac{x^2-3x+1}{x} \right) \left(\frac{x+8}{x^2+8x+1} \right) \\ &= \frac{x^2-3x+1}{x^2+8x+1}. \end{aligned}$$

Note that neither the top nor the bottom factor, so this is as far simplified as it can go.

5. (15 points) Multiply and simplify.

(a) (5 points) $\frac{x+1}{x+2} \cdot \frac{(x+2)(x+3)}{x+1}$

Solution: Cancel the $x+2$'s and $x+1$'s to get

$$\left(\frac{x+1}{x+2}\right) \left(\frac{(x+2)(x+3)}{x+1}\right) = x+3.$$

(b) (5 points) $\frac{(t+2)(4t+9)}{3t-2} \cdot \frac{6t-4}{(t-2)(4t+9)}$

Solution: Notice that $6t-4 = 2(3t-2)$. Cancel the factors of $3t-2$ and the factors of $4t+9$ to get

$$\begin{aligned} \left(\frac{(t+2)(4t+9)}{3t-2}\right) \left(\frac{6t-4}{(t-2)(4t+9)}\right) &= \left(\frac{t+2}{3t-2}\right) \left(\frac{2(3t-2)}{t-2}\right) \\ &= \frac{2t+4}{t-2}. \end{aligned}$$

(c) (5 points) $\frac{x^2-16}{x^2+7x+6} \cdot \frac{x+6}{x+4}$

Solution: Notice that

$$x^2-16 = (x-4)(x+4)$$

and

$$x^2+7x+6 = (x+6)(x+1).$$

Substitute in these factorizations and cancel appropriately to get

$$\begin{aligned} \left(\frac{x^2-16}{x^2+7x+6}\right) \left(\frac{x+6}{x+4}\right) &= \left(\frac{(x-4)(x+4)}{(x+6)(x+1)}\right) \left(\frac{x+6}{x+4}\right) \\ &= \frac{x-4}{x+1}. \end{aligned}$$

6. (10 points) Solve.

(a) (5 points) $\frac{4}{5} - \frac{1}{3} = \frac{x}{2}$

Solution: The common denominator on the left is 15 yielding

$$\frac{7}{15} = \frac{x}{2}.$$

Now multiply both sides by 2 to get

$$\frac{14}{15} = x.$$

(b) (5 points) $\frac{4}{x+1} + \frac{3}{x-5} = \frac{7}{x^2-4x-5}$

Solution: Notice that

$$x^2-4x-5 = (x-5)(x+1).$$

On the left the common denominator is $(x-5)(x+1)$ so write it in terms of that:

$$\frac{4(x-5)}{(x+1)(x-5)} + \frac{3(x+1)}{(x+1)(x-5)} = \frac{7}{(x+1)(x-5)}.$$

Simplify the left hand side to get

$$\frac{7x-17}{(x+1)(x-5)} = \frac{7}{(x+1)(x-5)}.$$

Now multiply both sides by $(x+1)(x-5)$ to get

$$7x-17 = 7.$$

Add 17 to get

$$7x = 7,$$

and finally divide by 7 to get $x = 1$.

7. (10 points) Do the following problems.

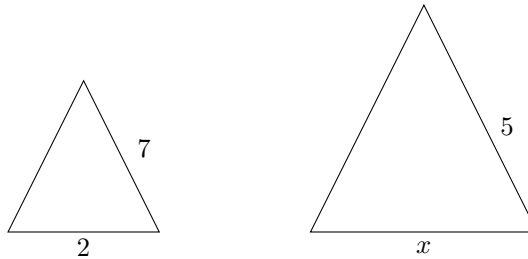
- (a) (5 points) Bob mows a lawn in 7 hours. It takes Mary 4 hours to mow the same lawn. How many hours does it take them working together?

Solution: The rate at which Bob mows the lawn is $\frac{1 \text{ lawn}}{7 \text{ hour}}$ and the rate at which Mary mows the lawn is $\frac{1 \text{ lawn}}{4 \text{ hour}}$. Therefore together they mow at a rate of

$$\left(\frac{1}{7} + \frac{1}{4}\right) \frac{\text{lawn}}{\text{hour}} = \frac{11 \text{ lawn}}{28 \text{ hour}}.$$

Therefore if they work together it should take $\frac{28}{11}$ hours to mow the one lawn.

- (b) (5 points) The following triangles are similar triangles. Find x .



Solution: Since the triangles are similar, we may write

$$\frac{2}{x} = \frac{7}{5}.$$

Therefore multiply by x to get

$$2 = \frac{7x}{5}.$$

Now multiply by 5 to get

$$10 = 7x,$$

and finally divide by 7 to get

$$x = \frac{10}{7}.$$