MATH 1011 - EXAM 3 - FALL 2016
SOLUTION

Thursday 13 October 2016
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (15 points) Factor completely.

   (a) (5 points) $24xy + 48x$
   Solution: Since 24 and x are common factors and $48 = 2 \cdot 24$, we factor them out and see

   $$24xy + 48x = 24x(y + 2).$$

   (b) (5 points) $6a^3b + 12a^2b^2$
   Solution: Since 6, a^2, and b are common factors and $12 = 6 \cdot 2$, we factor them out and see

   $$6a^3b + 12a^2b^2 = 6a^2b(a + 2b).$$

   (c) (5 points) $12x^3y + 2xy^3 + 6xy$
   Solution: Since 2, x, and y are common factors, factor them out and see

   $$12x^3y + 2xy^3 + 6xy = 2xy(6x^2 + y^2 + 3).$$

2. (15 points) Simplify and write with no negative exponent.

   (a) (5 points) 15^{-1}
   Solution: $\frac{1}{15}$

   (b) (5 points) $(2a^{-4}b^3)^2$
   Solution:

   $$(2a^{-4}b^3)^2 = 2^2\left(a^{-4}\right)^2\left(b^3\right)^2 = 4a^{-8}b^6 = \frac{4b^6}{a^8}.$$

   (c) (5 points) $\left(\frac{2x^2y^{-2}}{z}\right)^2\left(\frac{z^2}{x^2y^2}\right)^3$
   Solution:

   $$\begin{aligned}\left(\frac{2x^2y^{-2}}{z}\right)^2\left(\frac{z^2}{x^2y^2}\right)^3 &= \left(\frac{2^2\left(x^2\right)^2\left(y^{-2}\right)^2}{z^2}\right)\left(\frac{\left(z^2\right)^3}{\left(x^2\right)^3\left(y^2\right)^3}\right)\\ &= \left(\frac{4x^4y^{-4}}{z^2}\right)\left(\frac{z^6}{x^6y^6}\right)\\ &= 4x^{4-6}y^{-4-6}z^{6-2}\\ &= 4x^{-2}y^{-10}z^4\\ &= \frac{4z^4}{x^2y^{10}}.\end{aligned}$$

3. (5 points) (a) (2 points) Expand the scientific notation into a normal decimal: 2.63×10^{-6}
   Solution: 0.00000263

   (b) (3 points) Write using scientific notation: 0.000000000782
   Solution: 7.82×10^{-10}

4. (10 points) Factor by grouping.

   (a) (5 points) $x^2 + 7x + 2x + 14$
   Solution:

   $$\begin{aligned}(x^2 + 7x) + (2x + 14) &= x(x + 7) + 2(x + 7)\\ &= (x + 2)(x + 7)\end{aligned}$$

   (b) (5 points) $8x^2 + 6x + 4x + 3$
   Solution:

   $$\begin{aligned}(8x^2 + 6x) + (4x + 3) &= 2x(4x + 3) + (4x + 3)\\ &= (2x + 1)(4x + 3)\end{aligned}$$

5. (20 points) Factor completely or state that it is prime.

(a) (5 points) $x^2 + x - 6$

Solution: We seek numbers p and q so that

$$\begin{cases} p \cdot q = -6 \\ p + q = 1. \end{cases}$$

The numbers we need are $p = 3$ and $q = -2$. Now rewrite the original polynomial and factor by grouping to get

$$\left(x^2 + 3x\right) + ((-2)x + (-6)) = (x(x+3) + (-2)(x+3) = (x-2)(x+3).$$

(b) (5 points) $x^2 + x - 31$

Solution: Prime

(c) (5 points) $5x^2 - 9x - 2$

Solution: We need p and q so that

$$\begin{cases} p \cdot q = -10 \\ p + q = -9. \end{cases}$$

The numbers we need are $p = -10$ and $q = 1$. Now rewrite the original polynomial and factor by grouping to get

$$5x^2 - 9x - 2 = (5x^2 - 10x) + (x - 2) = 5x(x-2) + (x-2) = (5x+1)(x-2).$$

(d) (5 points) $x^4 - 16$

(hint: this is a difference of squares)

Solution: The easiest way is to use difference of squares factoring twice to factor

$$x^4 - 16 = (x^2 - 4)(x^2 + 4) = (x-2)(x+2)(x^2+4).$$

6. (10 points) Perform the long division.

(a) (5 points) $(x^2 + 3x + 2) \div (x + 2)$

Solution:

$$\begin{array}{r l} & \phantom{x^2+{}} x + 1 \\ x + 2 \big) & x^2 + 3x + 2 \\ & -x^2 - 2x \\ \hline & \phantom{x^2+{}} x + 2 \\ & -x - 2 \\ \hline & \phantom{x^2+x+{}} 0 \end{array}$$

(b) (5 points) $(3x^2 - 2x - 5) \div (x + 1)$

Solution:

$$\begin{array}{r l} & \phantom{3x^2-{}} 3x - 5 \\ x + 1 \big) & 3x^2 - 2x - 5 \\ & -3x^2 - 3x \\ \hline & -5x - 5 \\ & \phantom{3x^2-{}} 5x + 5 \\ \hline & \phantom{3x^2-5x-{}} 0 \end{array}$$

7. (15 points) Solve the equation.

(a) (5 points) $x(x+3) = 0$
Solution: Recall the zero-product property which says that if $A \cdot B = 0$ the n$A = 0$ or $B = 0$. Since this is already factored, we see $x = 0$ or $x + 3 = 0$, i.e. $x = -3$

(b) (5 points) $x^2 - 2x - 3 = 0$
Solution: Factor the left-hand side to get

$$(x-3)(x+1) = 0.$$

Using the zero-product property we get $x - 3 = 0$ or $x + 1 = 0$. Therefore $x = 3$ or $x = -1$.

(c) (5 points) $2x^2 + x - 6 = 0$
Solution: Factor the left-hand side to get

$$(2x-3)(x+2) = 0.$$

Using the zero-product property, we get $2x - 3 = 0$ or $x + 2 = 0$. Therefore $x = \frac{3}{2}$ or $x = -2$.

8. (10 points) Word problems.

(a) (5 points) The product of two consecutive numbers is 210. Find a pair of such numbers. (note: there are two pairs that could be found)
Solution: Let x be the first of the two consecutive numbers. Then the product of x and $x+1$ equaling 210 means

$$x(x+1) = 210,$$

or

$$x^2 + x - 210 = 0.$$

Factor the left-hand side to get

$$(x+15)(x-14) = 0.$$

Solve this by using the zero-product property: $x + 15 = 0$ or $x - 14 = 0$, therefore $x = -15$ or $x = 14$. Therefore the pair $-15, -14$ or the pair $14, 15$ obey the desired property.

(b) (5 points) The base of a triangle is 1 cm larger than its height. Its area is 6 cm^2. Find the length of the base and the height of the triangle.
Solution: Recall that $A = \frac{1}{2}bh$, where b represents the base of the triangle, h its height, and A its area. Plug in $A = 6$. We are told that $b = h + 1$, so the area equation becomes

$$6 = \frac{1}{2}(h+1)h.$$

Multiply by 2 to remove the fraction and distribute the h into the binomial to get

$$12 = h^2 + h.$$

Subtract by 12 to get

$$0 = h^2 + h - 12.$$

Factor the right-hand side to get

$$0 = (h-3)(h+4).$$

Now using the zero-product property, $h - 3 = 0$ or $h + 4 = 0$. Therefore $h = 3$ or $h = -4$. Since $h = -4$ is not physically meaningful, we are left with the solution $h = 3$. Since we saw earlier that $b = h + 1$, we get $b = 3 + 1 = 4$.