MATH 1011 - EXAM 2 FALL 2016
SOLUTION

Thursday 29 September 2016
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (5 points) Circle **T** for true and **F** for false.

   (a) (1 point) **T** (**F**) $5x^2 + 3\sqrt{x} + 1$ is a polynomial.
   Solution: The $\sqrt{x}$ that appears makes this not a polynomial.

   (b) (1 point) (**T**) **F** The ordered pair $(0, 1)$ is a solution of the equation $3y + 2x = 3$.
   Solution: Plugging in $x = 0$ and $y = 1$ into the equation $3y + 2x = 3$ yields $3(1) + 2(0) = 3$, which is true. Therefore $(0, 1)$ is a solution of $3y + 2x = 3$.

   (c) (1 point) **T** (**F**) The commutative law says that $a(b + c) = ab + ac$.
   Solution: This is the distributive law, not the commutative law.

   (d) (1 point) (**T**) **F** A polynomial with three terms is called a trinomial.
   Solution: This is true. A binomial has two terms, a trinomial has three terms, etc.

   (e) (1 point) **T** (**F**) A vertical line has zero slope.
   Solution: Horizontal lines have zero slope. Vertical lines have "undefined" or "∞" slope.

2. (10 points) Write the slope and the y-intercept of the lines described by the following equations.

   (a) (3 points) $y = 4x + 1$
   Solution: This equation is already in "slope-intercept" form $y = mx + b$ where m denotes the slope and b denotes the y-coordinate of the y-intercept (the y-intercept being a solution of the form $(0, b)$). Therefore
   Slope: 4 **y-intercept:** $(0, 1)$

   (b) (3 points) $2y = 6x + 8$
   Solution: This equation is not in "slope-intercept" form, so we will put it into this form. Divide by 2 to get

   $$y = 3x + 4.$$

   Now the equation is in slope-intercept form and we see
   Slope: 3 **y-intercept:** $(0, 4)$

   (c) (4 points) $3y - 6x = 9$
   Solution: This equation is not in "slope-intercept" form. To do this we solve for y in the equation. First add $6x$ to both sides to get

   $$3y = 6x + 9.$$

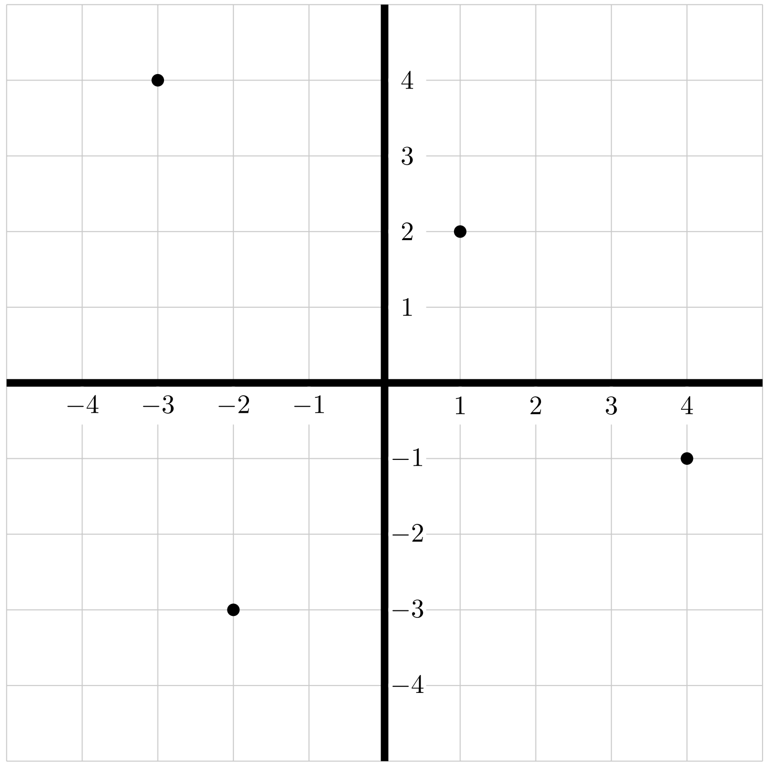
   Now divide by 3 to get

   $$y = 2x + 3.$$

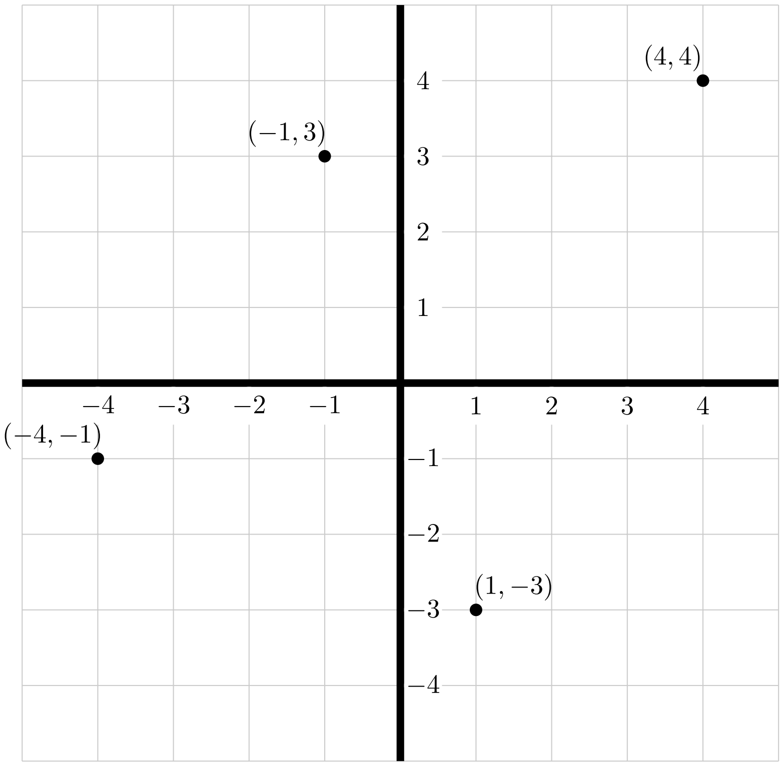
   Now we have put the equation into "slope-intercept" form, and so we may read off the slope and y-intercept.
   Slope: 2 **y-intercept:** $(0, 3)$

   NOTE: you could also find slope by finding two solutions (plugging in one value and solving for the other) and then computing the slope from the two points; you could also find the y-intercept by plugging in $x = 0$ and solving for y

3. (8 points) (a) (4 points) Plot the points $(1, 2)$, $(-3, 4)$, $(-2, -3)$, and $(4, -1)$ on the following coordinate plane:



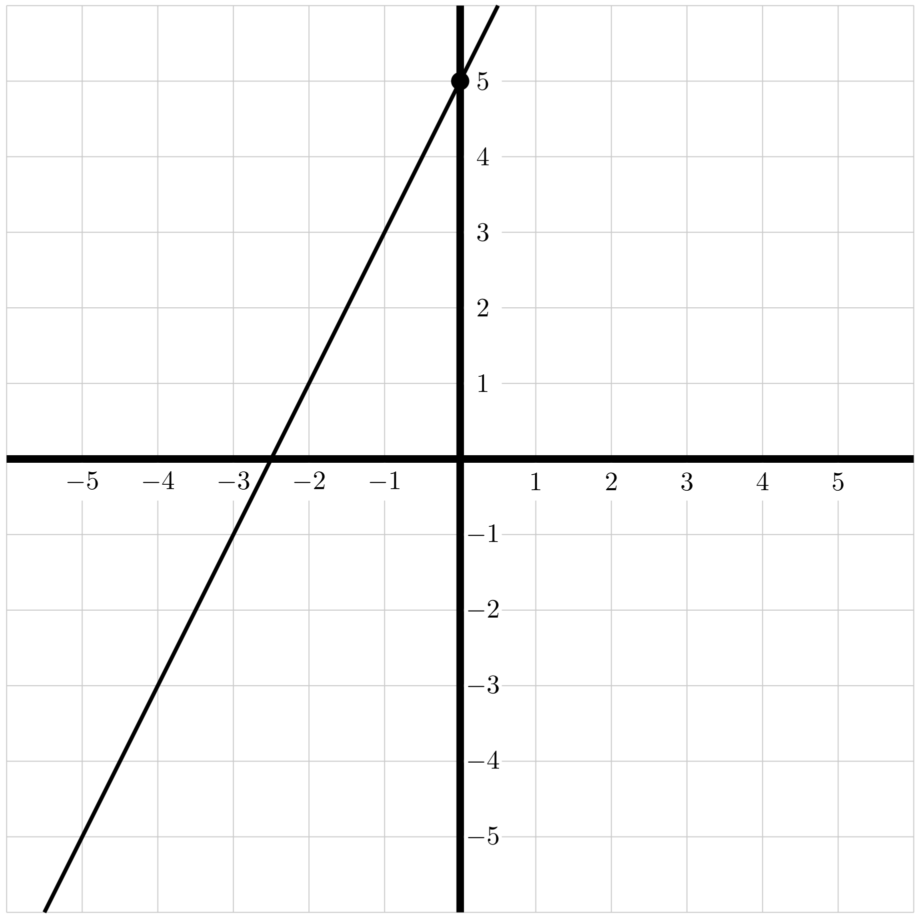
(b) (4 points) Write the coordinates of the points that appear in the following graph:



4. (12 points) Graph the line in any way you choose.

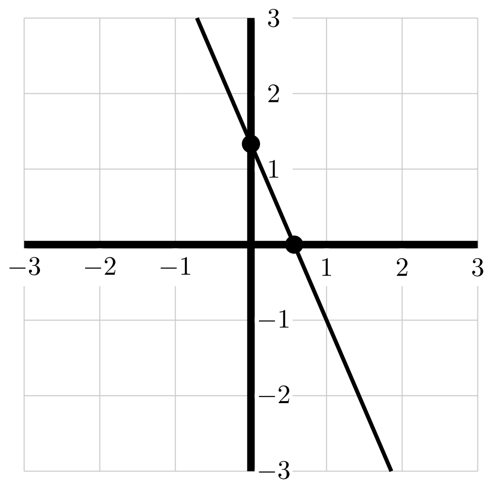
(a) (4 points) $y = 2x + 5$

Solution: This equation is in "slope-intercept" form and so it has slope 2 and y-intercept $(0, 5)$.

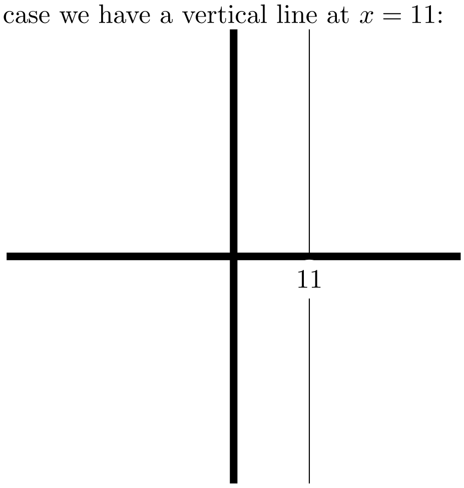


(b) (4 points) $3y + 7x = 4$

Solution: We could do this problem by putting it into "slope-intercept" form and reading off information. Instead we will find two solutions, plot them, and draw the line between them. First let $x = 0$ which leaves us with the equation $3y = 4$ which has solution $y = \frac{4}{3}$. This means $\left(0, \frac{4}{3}\right)$ is a solution. Now let $y = 0$ yielding the equation $7x = 4$, meaning $x = \frac{4}{7}$. Therefore $\left(\frac{4}{7}, 0\right)$ is a solution. Now plot these two points and draw the line between them:



(c) (4 points) $x = 11$
Solution: Recall that any equation of the form $x = a$ denotes a vertical line at $x = a$. In this case we have a vertical line at $x = 11$:



5. (15 points) You rent a car with a full tank of gas on 3 September at noon and you return the car on 8 September at noon. When the car was initially rented, the odometer read 5,120 miles and when you returned the car, the odometer read 5,720 miles. You returned the car with a full tank of gas and you had to put in a total of 20 gallons of gas over the whole trip. The whole rental cost you a total amount of $200 over the entire trip.

(a) (5 points) What was your average rate of travel, measured in $\frac{\text{miles}}{\text{day}}$?
Solution: Since you traveled for 5 days and you traveled $5,720 - 5,120 = 600$ miles, the average rate of travel is

$$\frac{600}{5}\frac{\text{miles}}{\text{day}} = 120\frac{\text{miles}}{\text{day}}.$$

(b) (5 points) What was the average gas mileage, measured in $\frac{\text{gallons}}{\text{mile}}$?
Solution: As seen above, you traveled 600 miles. Since you used 20 gallons of gas, your average gas mileage was

$$\frac{20}{600}\frac{\text{gallons}}{\text{mile}} = \frac{1}{30}\frac{\text{gallons}}{\text{mile}}.$$

Note: if we measured the gas mileage as $\frac{\text{miles}}{\text{gallon}}$ we would have gotten $30\frac{\text{miles}}{\text{gallon}}$. This would tell us how far each gallon of gas gets us, while the rate $\frac{\text{gallons}}{\text{mile}}$ tells us how much gas it takes go to one mile.

(c) (5 points) What was the average amount of money you spent per day, in appropriate units?
Solution: The units here would be $\frac{\text{dollars}}{\text{day}}$. Since you spent 200 dollars, you get

$$\frac{200}{5}\frac{\text{dollars}}{\text{day}} = 40\frac{\text{dollars}}{\text{day}}.$$

6. (15 points) (a) (5 points) Find an equation for the line with slope 2 and y-intercept $(0, 7)$.
Solution: From the slope-intercept form of the line, such a line would have equation

$$y = 2x + 7.$$

(b) (5 points) Find an equation of the line with slope 3 that passes through the point $(11, 13)$.
Solution: Using the point-slope form of the line we get

$$y - 13 = 3(x - 11).$$

(c) (5 points) Find and equation of the line that contains the points $(3, -4)$ and $(2, 1)$.
Solution: We need the slope. Compute it:

$$\text{slope} = \frac{1-(-4)}{2-3} = \frac{1+4}{2-3} = \frac{5}{-1} = -5.$$

Therefore using the point-slope form of the line and the point $(2, 1)$ we get

$$y - 1 = -5(x - 2).$$

7. (15 points) Simplify the following polynomial expressions completely.

(a) (5 points) $(3x^2 + 5x - 5) - (2x^2 + 4x - 3)$
Solution: Calculate

$$\begin{aligned}(3x^2 + 5x - 5) - (2x^2 + 4x - 3) \quad &= 3x^2 + 5x - 5 - 2x^2 - 4x + 3 \\ &= x^2 + x - 2.\end{aligned}$$

(b) (5 points) $(x + 3)(x + 1)$
Solution: Calculate

$$\begin{aligned}(x+3)(x+1) \quad &= (x+3)x + (x+3)(1) \\ &= x^2 + 3x + x + 3 \\ &= x^2 + 4x + 3.\end{aligned}$$

(c) (5 points) $(x^2 + 5x + 1)(2x^2 - x + 3)$
Solution: Calculate

$$\begin{aligned}(x^2 + 5x + 1)(2x^2 - x + 3) \quad &= (x^2 + 5x + 1)(2x^2) + (x^2 + 5x + 1)(-x) + (x^2 + 5x + 1)(3) \\ &= 2x^4 + 10x^3 + 2x^2 - x^3 - 5x^2 - x + 3x^2 + 15x + 3 \\ &= 2x^4 + 9x^3 + 0x^2 + 14x + 3 \\ &= 2x^4 + 9x^3 + 14x + 3\end{aligned}$$

8. (15 points) Simplify completely.

(a) (5 points) x^2x^3
Solution: Using the rule that $a^n a^m = a^{n+m}$ we see

$$x^2x^3 = x^{2+3} = x^5.$$

(b) (5 points) $(2x^2y^4)^2$
Solution: First we use the rule that powers distribute among a product to get

$$(2x^2y^4)^2 = 2^2(x^2)^2(y^4)^2.$$

Now we use the rule that $(a^n)^m = a^{nm}$ and so we continue the calculation:

$$(2x^2y^4)^2 = 4x^{2\cdot2}y^{4\cdot2} = 4x^4y^8.$$

(c) (5 points) $\left(\frac{xy}{z}\right)^3\left(\frac{3}{xy}\right)^2$

Solution: First we use the rule that powers distribute among a quotient to get

$$\left(\frac{xy}{z}\right)^3\left(\frac{3}{xy}\right) = \left(\frac{(xy)^3}{z^3}\right)\left(\frac{3^2}{(xy)^2}\right).$$

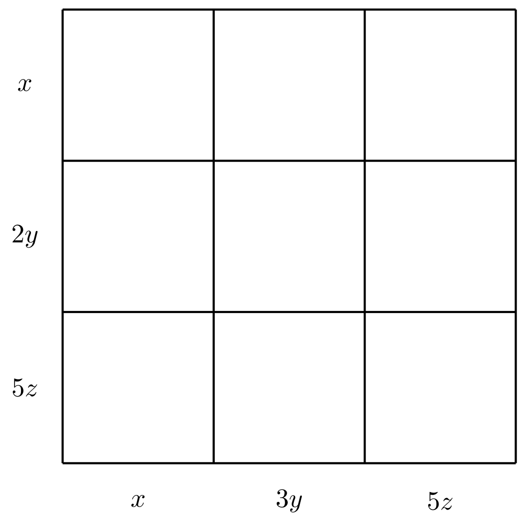
Using the law that powers distribute among products yields

$$\left(\frac{xy}{z}\right)^3\left(\frac{3}{xy}\right) = \left(\frac{x^3y^3}{z^3}\right)\left(\frac{9}{x^2y^2}\right).$$

Now using the law $\frac{a^n}{a^m} = a^{n-m}$, we get

$$\left(\frac{xy}{z}\right)^3\left(\frac{3}{xy}\right) = \frac{9}{z^3}x^{3-2}y^{3-2} = \frac{9xy}{z^3}.$$

9. (5 points) Write a polynomial that describes the area of the big rectangle:



Solution: Add up the areas of all the little squares:

$$x^2 + 3xy + 5xz + 2yx + 6y^2 + 10yz + 5zx + 15zy + 25z^2 = x^2 + 6y^2 + 25z^2 + 5xy + 10xz + 25yz.$$