

# MATH 1011 - EXAM 1 FALL 2016

Name: \_\_\_\_\_

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Instructor: Tom Cuchta

**Instructions:**

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (5 points) Circle **T** for true and **F** for false.

- (a) (1 point) **T**  **F** The associative law says that for all algebraic expressions  $a$  and  $b$ ,

$$a + b = b + a.$$

*Explanation: the commutative law says this. The associative law says that*

$$a + (b + c) = (a + b) + c.$$

- (b) (1 point)  **T** **F** The distributive law says that for all algebraic expressions  $a, b$ , and  $c$ ,

$$a(b + c) = ab + ac.$$

- (c) (1 point)  **T** **F** The number  $-\frac{5}{3}$  is a rational number.

*Explanation: Rational numbers are those numbers which are a quotient of integers. Since  $-5$  and  $3$  are integers,  $-\frac{5}{3}$  is rational.*

- (d) (1 point) **T**  **F** The number  $\pi$  is an integer.

*Explanation: we discussed in class that  $\pi$  is, in fact, an irrational number. No integer is an irrational number because all integers are rational.*

- (e) (1 point) **T**  **F** The number  $3$  is a solution of the equation

$$2x + 9 = -10000.$$

*Explanation: a solution of an equation is a value for the variable to take that makes the resulting equation true. If we substitute  $x = 3$  into the equation  $2x + 9 = -10000$  we get*

$$2 \cdot 3 + 9 = -10000,$$

*but the left hand side simplifies to  $6 + 9 = 15$ , and so the resulting equation is*

$$15 = -10000,$$

*which is not true. Therefore  $3$  is not a solution of this equation.*

2. (10 points) Evaluate the given expressions at the specified values.

- (a) (5 points)  $2x + 7w$  where  $x = 1$  and  $w = 2$

*Solution: If  $x = 1$  and  $w = 2$  we may [calculate](#)*

$$2 \cdot 1 + 7 \cdot 2 = 2 + 14 = 16.$$

- (b) (5 points)  $|-2x + w|$  where  $x = 2$  and  $w = -1$

*Solution: If  $x = 2$  and  $w = -1$  we may [calculate](#)*

$$|-2 \cdot 2 + 1| = |-4 + 1| = |-3| = 3.$$

3. (10 points) Find the prime factorization of the following numbers or state that they are prime.

- (a) (5 points)  $36$

*Solution: Draw a "factor tree" or simply [calculate](#) follows:*

$$36 = 2 \cdot 18 = 2 \cdot 2 \cdot 9 = 2 \cdot 2 \cdot 3 \cdot 3.$$

- (b) (5 points)  $13$

*Solution:  $13$  is a prime number.*

4. (15 points) Compute.

(a) (5 points)  $\frac{1}{2} + \frac{3}{5}$

*Solution:* The common denominator here is 10, meaning we will rewrite  $\frac{1}{2}$  as  $\frac{5}{10}$  and we will rewrite  $\frac{3}{5}$  as  $\frac{6}{10}$ . Then we **calculate**

$$\frac{1}{2} + \frac{3}{5} = \frac{5}{10} + \frac{6}{10} = \frac{11}{10}.$$

(b) (5 points)  $\frac{3}{4} \cdot \frac{5}{7}$

*Solution:* We simply multiply across and **calculate**

$$\frac{3}{4} \cdot \frac{5}{7} = \frac{3 \cdot 5}{4 \cdot 7} = \frac{15}{28}.$$

(c) (5 points)  $\frac{\frac{1}{2}}{\frac{2}{3}}$  *Solution:* Recall to evaluate a fraction of fractions we flip the bottom and multiply, i.e.  $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$ . Therefore we **calculate**

$$\frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} \cdot \frac{3}{2} = \frac{1 \cdot 3}{2 \cdot 2} = \frac{3}{4} \text{ reduce } \frac{1}{3}.$$

5. (5 points) Calculate the expression and completely simplify:

$$2 \cdot 3 + [1 - (2 \cdot 2 + 1)]^2.$$

*Solution:* Using the order of operations and taking it exactly one step at a time, we **calculate**

$$\begin{aligned} 2 \cdot 3 + [1 - (2 \cdot 2 + 1)]^2 &= 2 \cdot 3 + [1 - (4 + 1)]^2 \\ &= 2 \cdot 3 + [1 - 5]^2 \\ &= 2 \cdot 3 + [-4]^2 \\ &= 2 \cdot 3 + (-4)(-4) \\ &= 2 \cdot 3 + 16 \\ &= 6 + 16 \\ &= 22. \end{aligned}$$

6. (15 points) Solve the equation.

(a) (5 points)  $x - 1 = 4$

*Solution:* Add 1 to both sides to **get**

$$x = 5.$$

(b) (5 points)  $2x = 26$

*Solution:* Divide both sides by 2 to **get**

$$x = 13.$$

(c) (5 points)  $\frac{1}{2}x + \frac{1}{5} = \frac{3}{5}$

*Solution:* First subtract  $\frac{1}{5}$  from both sides to get

$$\frac{1}{2}x = \frac{2}{5}.$$

Now divide both sides by  $\frac{1}{2}$  to **get**

$$x = \frac{\frac{2}{5}}{\frac{1}{2}} = \frac{2}{5} \cdot \frac{2}{1} = \frac{4}{5}.$$

7. (10 points) Answer the following questions.

- (a) (5 points) What is 8% of 53? Give your answer as a decimal.

*Solution:* This question translates to the equation

$$x = 8 \cdot \frac{1}{100} \cdot 53.$$

Simplify the right hand side to [get](#)

$$x = \frac{8 \cdot 53}{100} = \frac{424}{100} = 4.24.$$

- (b) (5 points) What % of 31 is 7? (decimal not necessary)

*Solution:* This question translates to the equation

$$x \cdot \frac{1}{100} \cdot 31 = 7.$$

Simplify this to get

$$\frac{31}{100}x = 7.$$

Now divide both sides by  $\frac{31}{100}$  to get

$$x = \frac{7}{\frac{31}{100}} = \frac{7}{1} \cdot \frac{100}{31} = \frac{700}{31},$$

or in [other words](#),

$$x\% = \frac{7}{31} = 22.58\%.$$

8. (10 points) Do the following two problems.

- (a) (5 points) The relationship between the rate of travel (denoted  $r$ ), the distance traveled (denoted  $d$ ), and the time it took to travel (denoted  $t$ ) is

$$r = \frac{d}{t}.$$

If you traveled at a rate of  $50 \frac{\text{miles}}{\text{hour}}$  for 2 hours, how far did you travel?

*Solution:* We are given the information  $r = 50 \frac{\text{miles}}{\text{hour}}$  and  $t = 2$  hours. Plugging this information into the equation yields

$$50 \frac{\text{miles}}{\text{hour}} = \frac{d}{2 \text{ hours}}.$$

Therefore multiply both sides of the equation by 2 hours to get

$$\left(50 \frac{\text{miles}}{\text{hour}}\right) \cdot (2 \text{ hours}) = 100 \text{ miles}.$$

- (b) (5 points) The volume of a right circular cylinder with radius  $r$  and height  $h$  is

$$V = \pi r^2 h.$$

If a right circular cylinder has radius 1 inch and height 4 inches, what is its volume?

*Solution:* We are told that  $r = 1$  inch and  $h = 4$  inches. Substituting the values into the equation yields

$$V = \pi (1 \text{ inch})^2 (4 \text{ inches}) = 4\pi \text{ inches}^3.$$

9. (5 points) The sum of three consecutive numbers is 60. Find the numbers.

*Solution:* Let  $x$  be the first of the three numbers. Then  $x + 1$  is the second and  $x + 2$  is the third. Since we are told the sum of the three numbers is 60 we get the following equation:

$$x + (x + 1) + (x + 2) = 60.$$

Collect like terms on the left hand side to get

$$3x + 3 = 60.$$

Solve this equation by subtracting by 3 and then dividing by 3 to get

$$x = \frac{57}{3} = 19.$$

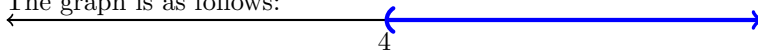
10. (15 points) Solve **and** graph the following inequalities.

- (a) (5 points)  $x - 1 > 3$

*Solution:* Add 1 to both sides to get

$$x > 4.$$

The graph is as follows:

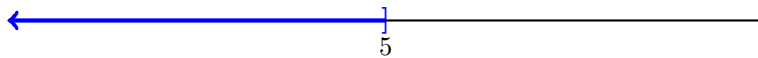


- (b) (5 points)  $2x - 3 \leq 7$

*Solution:* Add 3 and then divide by 2 to get

$$x \leq 5.$$

The graph is as follows:



- (c) (5 points)  $-2x + 5 < 2$

*Solution:* Subtract 5 to get

$$-2x < -3.$$

Divide by  $-2$  to get

$$x > \frac{-3}{-2} = \frac{3}{2}.$$

The graph is as follows:

