MATH 3304 Summer 2015
Homework 5 Name: ________________________

We have been studying differential equations of first and second order. This homework will introduce you to a related theory – difference equations. Difference equations is a theory where the "derivative" is replaced by the so-called "difference" operator Δ which is given by the formula

$$\Delta f(x) = f(x+1) - f(x).$$

1. (*2 points*) Let $f(x) = x^2 + 3x$. Calculate $\Delta f(x)$.

The Δ operator has many properties similar to the derivative, but slightly different. For example, a product rule for Δ is given by

$$\Delta[f(x)g(x)] = g(x+1)\Delta[f(x)] + f(x)\Delta[g(x)].$$

Very simply, a "difference equation" is the same thing as a differential equation, except instead of derivatives we use the Δ operator in its place. Let α be a constant. We define the "discrete exponential function" by the difference equation $\Delta y(x) = \alpha y(x); y(0) = 1$.

2. (*3 points*) Solve the difference equation $\Delta y(x) = \alpha y(x)$ by rewriting the left-hand-side using the definition of Δ and then solving the resulting equation for $y(x+1)$. Use this equation and the initial condition to determine values of the solution at $x = 1, 2, 3, 4$.

A popular notation for the solution found in Problem 2 is $e_\alpha(t)$; it obeys the formulas $\Delta e_\alpha(t) = \alpha e_\alpha(t)$ and $\Delta^2 e_\alpha(t) = \alpha^2 e_\alpha(t)$. With this information we can solve some second order homogeneous difference equations with constant coefficients by letting $y(t) = e_r(t)$, plugging it into the difference equation, and solving for r (just as we do for differential equations).

3. (*3 points*) Find the general solution of the second order difference equation

$$\Delta^2 y(t) - 4\Delta y(t) + 3y(t) = 0.$$