MATH 2222E Fall 2015
Quiz 2

SOLUTION

1. (*2 points*) If the following limit exists, compute it. If it does not exist, explain why not:

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^3}{x^2 + y^3}.$$

Solution: The limit does not exist. We will evaluate it along two different paths: the first path will be along the y-axis toward $(0,0)$ and the second path is along the x-axis toward $(0,0)$. Compute the limit along the first path:

$$\lim_{(0,y)\to(0,0)} \frac{0 - y^3}{0 + y^3} = \lim_{y\to 0} -1 = -1,$$

while the limit along the second path is

$$\lim_{(x,0)\to(0,0)} \frac{x^2 - 0}{x^2 + 0} = \lim_{x\to 0} 1 = 1.$$

Since the limit has a different value for two different paths, the limit $\lim_{(x,y)\to(0,0)} \frac{x^2 - y^3}{x^2 + y^3}$ does not exist.

2. (*3 points*) If the following limit exists, compute it. If it does not exist, explain why not:

$$\lim_{(x,y)\to(2,2)} \frac{2 - \sqrt{x+y}}{4 - x - y}.$$

Solution: This limit exists. To see this rationalize the numerator of the function in the limit to get

$$\frac{2 - \sqrt{x+y}}{4 - x - y}\left(\frac{2 + \sqrt{x+y}}{2 + \sqrt{x+y}}\right) = \frac{4 - x - y}{(4 - x - y)(2 + \sqrt{x+y})} = \frac{1}{2 + \sqrt{x+y}}.$$

Therefore

$$\lim_{(x,y)\to(2,2)} \frac{2 - \sqrt{x+y}}{4 - x - y} = \lim_{(x,y)\to(2,2)} \frac{1}{2 + \sqrt{x+y}} = \frac{1}{2 + \sqrt{4}} = \frac{1}{4}.$$