47) Verify $\frac{1-\cos x}{1+\cos x} = (\cot(x) - \csc(x))^2$

$\frac{(1-\cos x)}{(1+\cos x)}\left(\frac{1-\cos x}{1-\cos x}\right)$

$= \frac{1^2 - 2\cos x + \cos^2 x}{1-\cos^2 x}$

$= \frac{1 - 2\cos x + \cos^2 x}{\sin^2 x} = \csc^2 x - 2\cot(x)\csc(x) + \cot^2(x) = (\cot(x) - \csc(x))^2$ ✓

49) Verify $\frac{\cos(\theta)+1}{\tan^2(\theta)} = \frac{\cos(\theta)}{\sec(\theta)-1}$

$= \frac{\cos(\theta)}{\tan^2(\theta)} + \cot^2(\theta)$

$= \frac{\cos(\theta) + \cot^2(\theta)\tan^2(\theta)}{\tan^2\theta}$

$= \frac{\cos(\theta)+1}{\tan^2\theta} = \frac{\cos(\theta)+1}{?}$

51) Verify $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$

$\rightarrow \frac{1+\sin\theta + (1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$\rightarrow \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta} = 2\sec^2\theta$

53) $\frac{\cot(\alpha)+1}{\cot(\alpha)-1} = \frac{1+\tan\alpha}{1-\tan\alpha}$

55) $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

$(1+\tan^2 x)^2 - (1+\tan^2 x)$

$= 1 + 2\tan^2 x + \tan^4 x - 1 - \tan^2 x$

$= \tan^4 x + \tan^2 x$

57) $\frac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x} = \sec^2 x - \tan^2 x$

$= \frac{(\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)}{(\sec^2 x + \tan^2 x)}$

$= \sec^2 x - \tan^2 x$ ✓

77) $\sec(x) - \cos(x) + \csc(x) - \sin(x) - \sin(x)\tan(x) = \cos(x)\cot(x)$

$\Rightarrow \frac{1}{\cos(x)} - \cos(x) + \frac{1}{\sin x} - \sin(x) - \sin(x)\cdot\frac{\sin(x)}{\cos(x)}$

$= \frac{1-\cos^2 x}{\cos x} + \frac{1-\sin^2 x}{\sin x} - \frac{\sin^2 x}{\cos x}$

$= \frac{\sin^2 x - \sin^2 x}{\cos x} + \frac{\cos^2 x}{\sin x}$

$= 0 + \cos(x)\cot(x)$

$= \cos(x)\cot(x)$ ✓