The Train Running Problem

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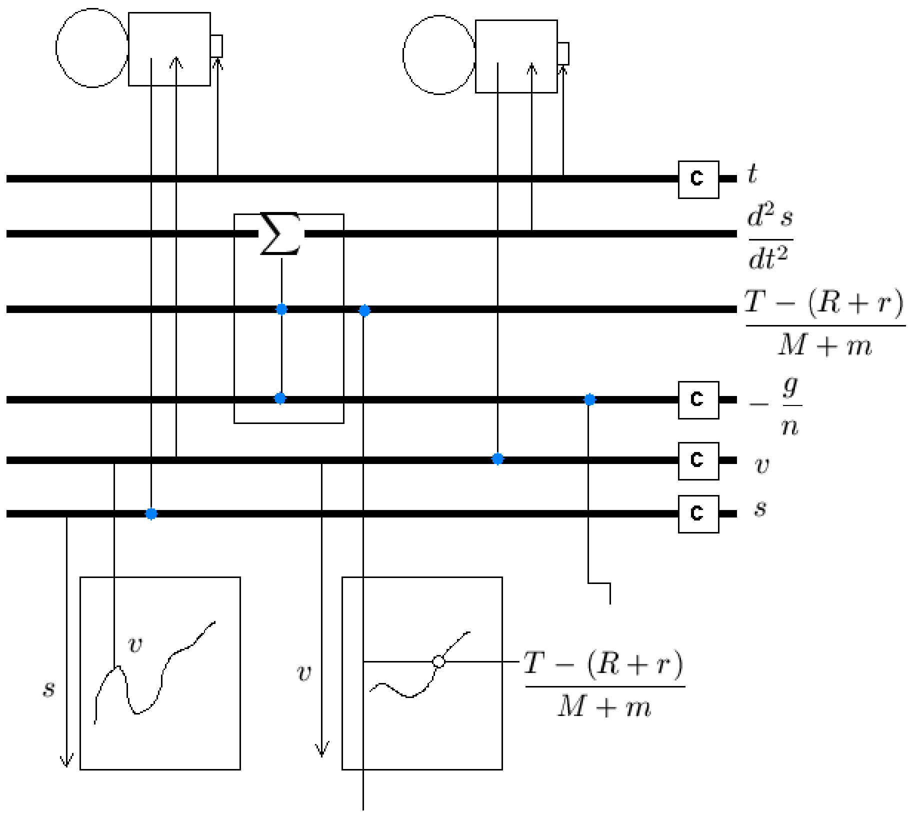
1 The Equation

The differential equation used in the train running problem is

$$\frac{dv}{dt} = \frac{T-(R+r)}{M+m} - \frac{g}{n} = \frac{d^2s}{dt^2}$$

The numerator of the first erm is a description of the orce that the train exerts on the rails (T = tractive force, R = force of engine, m = force of train) as a function of velocity; the denominator of that first term is the sum of the mass of the engine and the mass of the train. This division occurs because of Newton's second law $F = ma$ rearranged into the form $a = \frac{F}{m}$. The second term is an issue of the gradient of the ground the train is running on and its affect on acceleration: gravity multiplies with the gradient $\frac{1}{n}$ yields an acceleration term that can affect the total acceleraton of the train (going up a hill lowers acceleration, going down a hill increases it).

2 The Schematic



In order to graph velocity as a function of displacement, two integrators will be needed because the differential equation is given as the second derivative of displacement. So, since $\frac{d^2s}{dt^2} = \frac{T-(R+r)}{M+m} - \frac{g}{n}$, both terms of the equation must be summed by an adder in the machine. It should be noted that the terms themselves are not necessarily generated by any explicit continuous function that can be simply set into the machinery and let loose – they must be inputted with an input table by an operator because those terms make the differential equation nonlinear.

$\frac{T-(R+r)}{M+m}$ is a function of velocity, so the velocity shaft's motion must be carried to its input table to act as the independent variable. The gradient $\frac{g}{n}$ is a piecewise linear function of displacement, so the machine must keep track, with a counter, of where the train is at all times on the displacement shaft, and the machine must be turned off so the gradient can be modified properly in a piecewise manner (this is why the gradient term has no input table, it's unnecessary).

To plot the solution, simply tap off the displacement shaft as the independent variable and the velocity shaft as the dependent variable. This will plot the solution as a velocity function of displacement.