MTH 452 HW7

Sunday, March 31, 2024 9:59 PM

p.272: #1, 2, 3, 4, 5, 9, 10, 11, 12, 33, 34

(for grad: p.273: #29)

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p.272 #1 $\alpha = 1+\sqrt{2}$

$\alpha - 1 = \sqrt{2}$

subtract

$(\alpha-1)^2 = 2 \longrightarrow \alpha^2 - 2\alpha + 1 = 2 \rightarrow \alpha^2 - 2\alpha - 1 = 0$

$\Rightarrow f(x) = x^2 - 2x - 1 \in \mathbb{Q}[x]$

#2 $\alpha = \sqrt{2} + \sqrt{3}$

$\alpha^2 = (\sqrt{2}+\sqrt{3})^2$

$= 2 + 2\sqrt{6} + 3$

$\alpha^2 - 5 = 2\sqrt{6}$

$(\alpha^2-5)^2 = 4(6) = 24$

$\alpha^4 - 10\alpha^2 + 25 = 24$

$\Rightarrow f(x) = \alpha^4 - 10\alpha^2 + 1 \in \mathbb{Q}[x]$

and $f(\alpha) = 0$

#3 $\alpha = 1 + i$

$\alpha - 1 = i$

$(\alpha-1)^2 = i^2$

$\alpha^2 - 2\alpha + 1 = -1$

$\alpha^2 - 2\alpha + 2 = 0$

$\Rightarrow f(x) = x^2 - 2x + 2 \in \mathbb{Q}[x]$

$f(\alpha) = 0$

#4) $\alpha = \sqrt{1 + \sqrt[3]{2}}$

$\alpha^2 = 1 + \sqrt[3]{2}$

$\alpha^2 - 1 = \sqrt[3]{2}$

$(\alpha^2 - 1)^3 = 2$

$(\alpha^2 - 1)(\alpha^2 - 1) = \alpha^4 - 2\alpha^2 + 1$

$(\alpha^2 - 1)^3 = (\alpha^4 - 2\alpha^2 + 1)(\alpha^2 - 1)$

$= \alpha^6 - 2\alpha^4 + \alpha^2 - \alpha^4 + 2\alpha^2 - 1$

$= \alpha^6 - 3\alpha^4 + 3\alpha^2 - 1$

$\Rightarrow \begin{cases} f(x) = x^6 - 3x^4 + 3x^2 - 3 \in \mathbb{Q}[x] \\ f(\alpha) = 0 \end{cases}$

#5) $\alpha = \sqrt{\sqrt[3]{2} - i}$

$\alpha^2 = \sqrt[3]{2} - i$

$\alpha^6 = (\sqrt[3]{2} - i)^3 = (\sqrt[3]{2} - i)(\sqrt[3]{2} - i)(\sqrt[3]{2} - i)$

$= \left((\sqrt[3]{2})^2 - 2\sqrt[3]{2}\, i - i^2\right)(\sqrt[3]{2} - i)$

$= 2 - 2(\sqrt[3]{2})^2 i + \sqrt[3]{2} - (\sqrt[3]{2})^2 i - 2\sqrt[3]{2} - i$

Can't really do anything with this?

Classify $\alpha \in \mathbb{C}$ as algebraic or transcendental over F.

Classify $\alpha \in \mathbb{C}$ as algebraic or transcendental over F.

#9 $\alpha = i$, $F = \mathbb{Q}$

$\downarrow$

algebraic since

$$\underbrace{\alpha^2 + 1}_{\in \mathbb{Q}[\alpha]} = 0$$

#10 $\alpha = 1 + i$, $F = \mathbb{R}$

$$\alpha - 1 = i$$

$$\alpha^2 - 2\alpha + 1 = -1$$

$$\underbrace{\alpha^2 - 2\alpha + 2}_{\in \mathbb{R}[\alpha]} = 0$$

$\Downarrow$

algebraic

#11 $\alpha = \sqrt{\pi}$, $F = \mathbb{Q}$

$\downarrow$

transcendental

since π transcendental over $\mathbb{R}$ (also $\mathbb{Q}$)

#12 $\alpha = \sqrt{\pi}$, $F = \mathbb{R}$

$\downarrow$

transcendental