Quiz 6 MTH 450/550 Fall 2023

$*$	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

Find $\langle a \rangle = \{a^n : n \in \mathbb{Z}\}$

$a^2 = a * a = a$

$a^3 = a*(a*a) = a*a = a$

So, $\langle a \rangle = \{a\}$ (you see that a is the identity of $\langle G, * \rangle$!)

NOT a generator!

Find $\langle b \rangle$

$b^2 = b*b = C$

$b^3 = (b*b)*b = c*b = d$

$b^4 = ((b*b)*b)*b = d*b = a$

$b^5 = \cdots = a*b = b$

So, $\langle b \rangle = \{a, b, c, d\}$, i.e, b is a generator!

Find $\langle c \rangle$

$c^2 = c*c = a$

$c^3 = \cdots = a*c = c$

$c^4 = c*c = a$

$\vdots$

So $\langle c \rangle = \{a, c\}$, **NOT** a generator

Find $\langle d \rangle$

$d^2 = d*d = c$

$d^3 = \cdots = c*d = b$

$d^4 = \cdots = b*d = a$

$d^5 = \cdots = a*d = d$

So $\langle d \rangle = \{a, b, c, d\}$, i.e, d is a generator!