Quiz 12 MTH 450/550

Sunday, November 12, 2023 9:07 AM

$$\mathbb{Z}_3 \times \mathbb{Z}_4 = \left\{ \begin{matrix} (0,0), (0,1), (0,2), (0,3), \\ (1,0), (1,1), (1,2), (1,3), \\ (2,0), (2,1), (2,2), (2,3) \end{matrix} \right\}$$

$\langle (0,0) \rangle = \{(0,0)\}$

$\langle (0,1) \rangle = \{(0,1), (0,2), (0,3), (0,0)\}$

$\langle (0,2) \rangle = \{(0,2), (0,0)\}$

$\langle (0,3) \rangle = \{(0,3), (0,2), (0,1), (0,0)\}$

$2 = 6 \bmod 4 \qquad 1 = 5 \bmod 4$

$\langle (1,0) \rangle = \{(1,0), (2,0), (0,0)\}$

$\langle (1,1) \rangle = \{(1,1), (2,2), (0,3), (1,0), (2,1), (0,2), (1,3), (2,0), (0,1), (1,2), (2,3), (0,0)\}$

equals the whole group $\mathbb{Z}_3 \times \mathbb{Z}_4$

$\Rightarrow \mathbb{Z}_3 \times \mathbb{Z}_4$ is cyclic

$\Rightarrow \mathbb{Z}_3 \times \mathbb{Z}_4$ isomorphic to $\mathbb{Z}_{12}$