HW9 MTH450/550 Fall 2023

Sunday, November 12, 2023   3:47 PM

pg. 83 #11 $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$

Orbit of 1: $\{1,3,4,5,6,2\}$

#12 $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$

orbit of 1:

$\{1,2,4,3\}$

#13 $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$

Orbit of 1: $\{1,5\}$

p.94 #1 $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 3 & 6 & 2 & 4 \end{pmatrix}$

orbit of 1: $\{1,5,2\}$

orbit of 3: $\{3\}$

Orbit of 4: $\{4,6\}$

all numbers accounted for

#3 $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 5 & 1 & 4 & 6 & 8 & 7 \end{pmatrix}$

orbit of 1: $\{1,2,3,5,4\}$

orbit of 6: $\{6\}$

orbit of 7: $\{7,8\}$

#7 $(1,4,5)(7,8)(2,5,7) = \begin{pmatrix} 1&2&3&4&5&6&7&8 \\ 4&2&3&5&1&6&7&8 \end{pmatrix}\begin{pmatrix} 1&2&3&4&5&6&7&8 \\ 1&2&3&4&5&6&8&7 \end{pmatrix}\begin{pmatrix} 1&2&3&4&5&6&7&8 \\ 1&5&3&4&7&6&2&8 \end{pmatrix}$

$= \begin{pmatrix} 1&2&3&4&5&6&7&8 \\ 4&1&3&5&8&6&2&7 \end{pmatrix}$

#10

$\begin{pmatrix} 1&2&3&4&5&6&7&8 \\ 8&2&6&3&7&4&5&1 \end{pmatrix} = (1,8)(3,6,4)(5,7)$ ← disjoint cycles

transpositions

not a transposition

orbits

$\{1,8\},\{2\},\{3,6,4\},\{5,7\}$

Write $(3,6,4)$ as transpositions: $(3,4)(3,6)$

So

$y = (1,8)(3,4)(3,6)(5,7)$

#31 A infinite. $H = \{\sigma \in S_A : \sigma$ moves at most finitely many elts$\}$

Is H a subgroup of S_A?

Yes: ① if $\sigma, \tau \in H$, then $\sigma\tau \in H$ since it can only move finitely many elts since that's what σ and τ do $\Rightarrow$ closed under op

② identity permutation moves 0 elts, which is finite, so the identity permutation is in H $\Rightarrow$ contains identity

③ inverse of a permutation that moves only finitely many elts would move those finitely many elts back

$\Rightarrow$ closed under inverses

p.101 #1 cosets of $4\mathbb{Z}$ in $\mathbb{Z}$:

all the cosets

$4\mathbb{Z} = \{\dots,-8,-4,0,4,8,\dots\}$

$1(4\mathbb{Z}) = \{\dots,-7,-3,1,5,9,\dots\}$

$2(4\mathbb{Z}) = \{\dots,-6,-2,2,6,10,\dots\}$

$3(4\mathbb{Z}) = \{\dots,-5,-1,3,7,11,\dots\}$

$4(4\mathbb{Z}) = \{\dots,-4,0,4,8,12,\dots\}$

same

#2 cosets of $4\mathbb{Z}$ in $2\mathbb{Z}$

all the cosets

$4\mathbb{Z} = \{\dots,-8,-4,0,4,8,\dots\}$

$2(4\mathbb{Z}) = \{\dots,-6,-2,2,6,10,\dots\}$

$4(4\mathbb{Z}) = \{\dots,-4,0,4,8,12,\dots\}$

same

#3 cosets of $\langle 2 \rangle$ in $\mathbb{Z}_{12}$

all the cosets

$\langle 2 \rangle = \{2,4,6,8,10,0\}$

$1\langle 2 \rangle = \{3,5,7,9,11,1\}$

$2\langle 2 \rangle = \{4,6,8,10,0,2\}$

same