HW2 MTH 450-550 Fall 2023

p.8 #24 Partitions of $\{a,b\}$:

$\{\{a\},\{b\}\}$ and $\{\{a,b\}\}$

and no others. So, 2

p.8 #25

Partitions of $\{a,b,c\}$:

$\{\{a\},\{b,c\}\}$, $\{\{a,b\},\{c\}\}$, $\{\{a,b,c\}\}$

$\{\{b\},\{a,c\}\}$, and $\{\{a\},\{b\},\{c\}\}$

So, 5

p.8 #30 not an equiv relation:

for example $\underbrace{5 \geq 3}_{\text{true}}$ but $3 \geq 5$ is false

so it fails symmetry

p.8 #31 yes an equiv relation

partition arising: the cells are

See Thm 0.22

$$\bar{a} = \{x \in \mathbb{R} : |x| = |a|\}$$
$$= \{a, -a\}$$

So the partition is

$$\{\bar{a} : a \in \mathbb{R}\} = \{\{a,-a\} : a \in \mathbb{R}\}$$

p.8 #35 a) $\{2,4,6,8,10,\ldots\}$, $\{1,3,5,7,\ldots\}$

b) $\{1,4,7,10,\ldots\}$, $\{2,5,8,\ldots\}$, $\{3,6,9,\ldots\}$

c) $\{1,6,11,16,\ldots\}$, $\{2,7,12,\ldots\}$, $\{3,8,13,\ldots\}$, $\{4,9,14,\ldots\}$, $\{5,10,15,\ldots\}$

p.25 #1

$b * d = e$

$c * c = b$

$[(a*c)*e]*a = [c*e]*a$
$= a*a = a$

p.25 #2

$(a*b)*c = b*a = b$

$a*(b*c) = a*a = a$

not associative since these aren't equal

p.25 #3

$(b*d)*c = e*c = a$

$b*(d*c) = b*b = c$

not associative

p.25 #4

No, $b*e = c$
$e*b = b$ not equal

p.25 #5

$*$	a	b	c	d
a	a	b	c	a
b	b	d	a	c
c	c	a	d	b
d	d	c	b	a

p.25 #6

★	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	c	d	c	d
d	d	d	d	d

Need...

$$a\star(d\star b) = \underbrace{(a\star b)}\star d$$
$$= b\star d$$
$$= d$$

Need

$$a\star(d\star b) = d$$

So what x solves

$$a\star x = d\ ?$$

$\downarrow$

$\boxed{x=d}$ only choice

So we require $\boxed{d\star b = d}$

Need...

$$b\star(d\star c) = \underbrace{(b\star d)}\star c$$
$$= d\star c$$

$\downarrow$

works if $\boxed{d\star c = d}$

Need...

$$c\star(d\star a) = (c\star d)\star a$$
$$= d\star a$$

$\downarrow$

works if $\boxed{d\star a = d}$

Need...

$$d\star(d\star d) = (d\star d)\star d$$

$\downarrow$

works if $d\star d = d$.

p.25 #9

Commutative: $a\star b = \frac{ab}{2} = \frac{ba}{2} = b\star a$ ✓

associative: $a\star(b\star c) = a\star\left(\frac{bc}{2}\right) = \frac{abc}{4} = \left(\frac{ab}{2}\right)\star c = (a\star b)\star c$ ✓

p.25 #10

Comm: $a\star b = 2^{ab} = 2^{ba} = b\star a$ ✓

assoc: $a\star(b\star c) = a\star 2^{bc} = 2^{(a)(2^{bc})}$

$(a\star b)\star c = 2^{ab}\star c = 2^{(2^{ab})(c)}$

not equal! $\rightarrow$ not assoc

p.25 #18 yes bin op

p.25 #19 yes bin op

p 8 # 28 skip

p.8 #32 not equiv rel
not transitive b/c

$x=-2,\ y=0,\ z=2$

we have

$\underbrace{|x-y|=2\leq 3}_{x\mathcal{R}y}$ and $\underbrace{|y-z|=2\leq 3}_{y\mathcal{R}z}$

but $x\mathcal{R}z$ is false since

$|x-z|=|-4|=4\leq 3$ is false

p.8 #33 yes equiv relation

portions : $\{1,2,\ldots,9\}$
$\{10,11,\ldots,99\}$
$\{100,101,\ldots,999\}$
$\{1000,1001,\ldots,9999\}$
$\vdots$

p.25 #12 skip

p.8 #36

a) symmetric: if $r\sim s$, that means
$n \mid r-s$
"divides"
Since n would also divide $s-r$,
we get $s\sim r$. ✓

reflexive : $r\sim r$ implies $q=0$ in
$r-r=nq$ ✓

transitive if $r\sim s$ and $s\sim t$, then
$\exists q_1$ s.t. $r-s=nq_1$ and $\exists q_2$ s.t. $s-t=nq_2$

add

$r-t=nq_1+nq_2$
$=n(q_1+q_2)$

Since $q_1+q_2\in\mathbb{Z}$, we are done.

p.25 #11

comm : $a*b=a^b$
$b*a=b^a$ not equal in general

$\Rightarrow$ not comm

assoc : $a*(b*c)=a^{b*c}=a^{b^c}$ not equal
$(a*b)*c=a^b*c=(a^b)^c=a^{bc}$

(note : a^{b^c} is different than $(a^b)^c$ ~ see
for ex
$(2^3)^2=2^6=64$
while
$2^{3^2}=2^9=512$)